

DIAGNOSIS OF INDIVIDUAL DIFFICULTIES IN ARITHMETIC

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PREFACE

RESULTS of educational research constantly emphasise the nature of individual differences in children's mental powers, in their learning rates and in the levels of their scholastic attainments, but it is doubtful if practical teaching has been sufficiently influenced by these findings. Although methods and materials have improved in most school subjects, yet there is need for greater consideration of pupils' individual and particular requirements. Nowhere is this need so urgent as in arithmetic with its countless steps, its varied material and its numerous difficulties.

It is the primary purpose of this book to assist teachers in the diagnosis and remedying of pupils' difficulties in arithmetic. The corrective and remedial aspects of this subject form a very important part of the primary school teacher's everyday work, for arithmetic still looms largely in the school curriculum and it is still the stumbling-block of many pupils. Once a pupil has experienced difficulty in arithmetic it is essential that he should receive some individual help, whether his handicap be a minor or a major one. Very often the right kind of help given in the early stages prevents confusion in the later stages and minimises the possibility of backwardness. Moreover, it should be remembered that progress in arithmetic is as much dependent upon emotional as upon intellectual factors and that individual consideration of pupil difficulties is the best means of

dispersing the psychological effects of failure. But if individual aid is to be fully effective it must be given systematically and produce positive results, and this is only possible where diagnosis has been both accurate and comprehensive.

It was to provide a means of systematic survey and accurate diagnosis of difficulties in arithmetic that the Schonell Diagnostic Arithmetic Tests were constructed. The Tests are so compiled that they cover all the basic number combinations in the four processes and all the important steps in each process, and hence use of them indicates exactly every unit or step in which pupils are failing. This knowledge, combined with information on causation of errors and reasons for backwardness in arithmetic, enables teachers to distribute both attention and remedial practice to maximum advantage. The element of chance in arithmetic teaching is thereby largely eliminated : the haphazard discovery of inaccurate knowledge is replaced by scientific analysis of achievements in all fundamentals. Such an analysis not only means that effective assistance can be given to pupils by use of carefully graded exercises, but it also reveals a veritable mine of information which can be utilised to frame positive teaching methods to prevent backwardness.

The author wishes to express his gratitude to children and teachers who have contributed towards the preparation and standardisation of the Diagnostic Arithmetic Tests, and especially to Head-teachers of numerous schools of the London County Council. In particular he wishes to thank Dr S. H. Cracknell, M.Sc., Mr J. B. Third and Mr L. G. Lovell for help rendered from time to time. He is indebted to Mr R. G. Adams, B.Sc., M.A., for assistance with marking and calculations. He wishes, too, to thank Mr D. Munro, M.A., LL.B.,

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Above all he wishes to thank his wife for constant encouragement and continuous constructive assistance with the entire material.

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PREFACE TO THE SECOND EDITION

THE demand for a second edition of this small book is a gratifying indication of the growing use of diagnostic tests for the more scientific ascertainment of pupils' difficulties in arithmetic. Teachers, and educators in general, are realising the extreme importance of sound foundations in arithmetic and of the need for consideration of the individual pupil to prevent far-reaching emotional handicaps. Although there are still a few teachers using text-books which require pupils of 9+ to find the number of minutes, for example, in 3 years 2 months 3 weeks 6 days 18 hours, yet most of them show psychological understanding in the use of scientifically planned pupil texts and carefully graded remedial material.

Apart from a few minor alterations the only additions to this revised edition of the book are the average numbers of sums for Tests 6 to 12 in the *Diagnostic Arithmetic Tests* correctly worked in given times by age groups 7+ to 14 years. These averages (on page 100) will be of decided value in providing teachers with estimates of levels reached, and of progress achieved, by their pupils in the fundamental processes.

F. J. S.

SPECIAL NOTE ON NORMS (PAGES 48-57)

It is most important to note that the age given in these tables refers to the middle of the year mentioned and not the beginning. Thus a tabulated age of "8" means a strict age of 8 years 6 months, NOT 8 years 0 months, as might be thought.

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CHAPTER I

NATURE AND PURPOSE OF DIAGNOSTIC TESTS

THE wide variation in arithmetic attainments in the primary school is due to several factors. Firstly, ability to succeed in arithmetic, particularly mental and problem, is dependent upon at least a normal degree of general intelligence, that is on the ability to see relationships and to apply them to new situations. Thus, as there is a wide range in innate intellectual power amongst pupils, it follows that there will be a wide range in arithmetical ability, irrespective of any other conditioning factors.

Secondly, arithmetic is strikingly susceptible to the influence of emotional attitudes and tendencies. Here again, as there is a wide range in degrees of general emotional stability and in the nature of particular temperamental attitudes amongst pupils, so invariably we find that there are pronounced variations in arithmetical attainments if we consider only the emotional equipment of our pupils in relation to their success in arithmetic lessons. Pupils who are nervous, highly strung, unstable or lacking in persistence or concentration show their handicaps soonest and most markedly in arithmetic. A child needs only to be upset for a minute or to day-dream for a moment during an arithmetic lesson for his work soon to show the influence of his temporary lapses from stability and concentration. Such lapses

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may not be important in, say, reading or written composition, but in arithmetic, where final accuracy depends on accuracy in each preceding step, then persistent application is necessary on the part of the pupil.

Thirdly, arithmetic with its manifold types of sums, each requiring detailed teaching in the initial stages and carefully planned practice in later stages, is a subject in which the pupil loses undue ground from absences whether they be intermittent or continuous. Hence we find that, owing to a combination of factors, some innate, some environmental, pupils within a school-class present to the teacher a great diversity of individual difficulties in arithmetic. Naturally division of the class into sections helps considerably, but even then, with simpler material and with less expected from the more backward pupils, there are still many who flounder because of individual obstacles which have never been satisfactorily surmounted. It is in the full detection of these difficulties that not only the pupil, but also the teacher, requires assistance. A teacher knows the levels of his respective pupils in arithmetic : that John is weak in subtraction, that Lily makes errors in addition and that Ben invariably makes mistakes in multiplication ; but what he requires to know exactly is the nature, extent and cause of the pupils' errors in these particular processes. He can then reduce the extent of failure amongst his class by distributing his time and suitable remedial work more effectively.

HOW THE PURPOSE IS ACHIEVED

If we consider arithmetical attainments from both a qualitative and a quantitative standpoint we can distinguish four main criteria, accuracy, speed of

working, methods of work and extent of arithmetical processes mastered. Obviously some of these essentials are relatively more important than others ; their values differ with pupils of different ages and different mental calibres. For our normally intelligent pupils who will pass to some form of selective post-primary education all four criteria are important ; with duller pupils accuracy is the outstanding necessity, but speed, methods of working and range of processes known also need consideration. It is obvious that a class teacher obtains through the medium of his class-work and through weekly or monthly tests an indication of his pupils' ability in each of the four directions. But this is not enough for teaching purposes, particularly with those pupils who are backward in their work ; with them he requires a more analytic estimate of their achievements. In this he can obtain help from two sources :

- (a) The use of diagnostic tests.
- (b) A knowledge of common errors in the fundamental processes.

By these two aids he can *achieve the purpose*, not only of diagnosing difficulties, but of apportioning suitable remedial material on scientific lines and of preventing further errors.

(a) *Diagnostic Tests*

A diagnostic test in any school subject is constructed for the specific purpose of analysing the exact nature of the progress made by pupils in each important aspect of the subject. The test takes into consideration all the vital skills involved in each important aspect and these are tested by a series of carefully graded examples which cover all important steps in the acquirement of the skill. Thus a diagnostic test differs from an ordinary class-room test or

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from a standardised scholastic test in so far as its main object is to analyse not to assess. It differs too in neglecting the speed factor, for in a diagnostic test ample time is provided for pupils to complete all they are able to do. Differences between diagnostic and the usual standardised attainment tests are made clear if we consider examples of tests in a particular aspect of a particular subject.

Thus in arithmetic, in one standardised subtraction test¹ the pupils are given 5 minutes in which to work as many as possible of this type of subtraction exercise :

9802	7721	4944	3208	5381
6246	1841	1295	1738	3676

The emphasis in this instance is on speed and accuracy in an advanced form of subtraction and the level attained by the pupil can be estimated from a table of averages devised from application of the test to large numbers of pupils of different ages. Another type of subtraction test² consists of nineteen exercises, including subtraction of weights and measures, for which 7 minutes is allowed.

A further form of subtraction test,³ which is particularly applicable to young, and to dull and backward children, is that compiled and standardised by Sleight. In this, more attention has been paid to grading and the test consists of forty examples ranging in difficulty from 5 minus 2 to 7572 minus 3724. A time limit of 3 minutes is set.

Although all of these tests of subtraction enable a

¹ C. Burt, *Mental and Scholastic Tests*, pp. 300, 302 and 367.
P. S. King & Son.

² *Northumberland Standardised Tests. Arithmetic*. Compiled by C. Burt. University of London Press.

³ G. F. Sleight, *Diagnosis and Treatment of the Dull and Backward Child*, p. 272. Ph.D. Thesis in the University of London Library.

general estimate of speed and accuracy in subtraction to be made by means of the carefully compiled averages for pupils of different ages, yet none of them are fully diagnostic in the sense that they present all possible steps in subtraction, arranged in order of difficulty, so that the exact level of the pupil's mastery of subtraction and the exact location of his weaknesses can be discovered. It is just this objective that characterises the diagnostic test; it aims at gathering information on all aspects of the subtraction process, irrespective of the speed factor, at finding out just what the pupil can do and the precise step at which his knowledge breaks down.

Hence in a diagnostic test in subtraction the first sub-test consists of the 100 basic combinations of numbers under 20, which are essential to all subtraction exercises, for example :

$$\begin{array}{ccccc}
 9 - 2 & 9 - 4 & 10 - 5 & 7 - 0 & 8 - 5 \\
 11 - 2 & 10 - 6 & 12 - 8 & 14 - 7 & 12 - 9 \\
 8 - 3 & 10 - 8 & 11 - 5 & 10 - 9 & 11 - 8
 \end{array}$$

The pupils work all combinations and from the results the teacher can tell what degree of mastery of these fundamental facts has been attained by each pupil. He learns what are the individual failings and whether they are numerous or isolated; and he learns what, if any, are the failings of the class as a whole. With this information before him he can determine whether or not the pupils can reasonably be expected to succeed with more difficult examples.

The next sub-test is based on an adequate sampling of all the vital steps in subtraction in their order of difficulty. Four examples are allotted to each step. The first step is quite simple subtraction in which there is no "borrowing,"¹ there are only

¹ The term "borrowing" is used throughout the book in inverted commas. It is not meant to refer in any way to the

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tens and units in the minuend and units in the subtrahend, thus :

$$\begin{array}{r} 98 \\ - 3 \\ \hline \end{array} \qquad \begin{array}{r} 57 \\ - 4 \\ \hline \end{array}$$

In the next step there are tens and units in both minuend and subtrahend, but no "borrowing," thus :

$$\begin{array}{r} 55 \\ - 32 \\ \hline \end{array} \qquad \begin{array}{r} 99 \\ - 43 \\ \hline \end{array}$$

while the third step extends this form of subtraction, i.e. without "borrowing" to three figures in both minuend and subtrahend, thus :

$$\begin{array}{r} 346 \\ - 215 \\ \hline \end{array} \qquad \begin{array}{r} 987 \\ - 832 \\ \hline \end{array}$$

Step four involves this form of example :

$$\begin{array}{r} 18 \\ - 14 \\ \hline \end{array} \qquad \begin{array}{r} 16 \\ - 10 \\ \hline \end{array}$$

i.e. a simple introduction to "0" difficulties.

The next step introduces the pupil to simple "borrowing," but with a unit figure only, in the subtrahend, thus :

$$\begin{array}{r} 71 \\ - 2 \\ \hline \end{array} \qquad \begin{array}{r} 62 \\ - 4 \\ \hline \end{array}$$

This is then extended by a very small increase in difficulty to the form in which there is "borrowing"

decomposition method of subtraction but to the equal addition method, and as a convenient term covering the step of adding ten to the minuend and adding a compensating ten to the subtrahend. Were there another suitable term which would connote adequately the rather long verbal explanation in the equal addition method it would have been used. It is assumed, however, that all teachers now employ the equal addition method (certainly the process which produces greater accuracy and speed), and therefore the term "borrowing" can still stand for the idea of adding ten to both lines.

only in the units column, but there are two figures in both minuend and subtrahend, for example :

$$\begin{array}{r} 54 \\ - 39 \\ \hline \end{array} \qquad \begin{array}{r} 22 \\ - 17 \\ \hline \end{array}$$

In this manner the diagnostic subtraction test proceeds by a further eight steps, each involving a slight increase in difficulty until the final example,

$$\begin{array}{r} 6067 \\ - 5970 \\ \hline \end{array}$$

represents a high-water mark in subtraction ability.

In preceding paragraphs we have examined two diagnostic sub-tests in subtraction, but naturally a complete battery of such tests would include a similar consideration of other processes, addition, multiplication and division. A further extension would include graded material in fractions, decimals and percentages. It is quite clear from our detailed consideration of the diagnostic tests in subtraction that the teacher will obtain from the use of such tests much valuable information of an exceedingly systematised and individualised kind. The results will reveal, at one testing and in a comprehensive way, the *exact* level reached by his pupils and the *precise* nature of their difficulties. There will be no hit or miss about the examination and nothing will be left to chance ; the teacher will realise that instead of having to discover at random the subtraction difficulties of his pupils through their everyday work—a very unsatisfactory method—he has in the results of the diagnostic test a complete inventory of their subtraction attainments.

Similarly the diagnostic tests in the other processes will discover for him the pupils' exact equipment in addition, multiplication and division. And

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it must be borne in mind that automatic accuracy in the basic combinations and in the simpler forms of the fundamental processes is the foundation of all arithmetic.

(b) *Knowledge of Common Errors in the Four Rules*

By far the most valuable approach to the diagnosis of difficulties in arithmetic is made by using a carefully constructed diagnostic test, but results from these can be made even more useful if they are supplemented by information on the pupil's methods of working and on the reasons for their errors. In many cases this information can be derived from a scrutiny of the pupils' work, but there are some instances, particularly with pupils who are very backward in arithmetic, where it is necessary to make observation of their arithmetical habits and to employ oral analysis of their written work.

The most useful way of obtaining an insight into the pupil's arithmetical methods is to ask him to work aloud a number of significant examples from the diagnostic tests, including steps where he has made errors. Methods of working, especially in addition, but also in all sums where more difficult number combinations are involved, should be noted. Here a word of warning is necessary. The atmosphere in the class-room during arithmetic is of vital importance and the teacher must see that this oral analysis does not become oral inquisition. There are some teachers who do too much working aloud of sums with their backward pupils ; this is bad, not only because it deprives such pupils of the very independence and initiative in arithmetic which the teachers are seeking to develop, but because some children, frequently nervous ones and those labouring under inferiority attitudes, make more mistakes when all their working is done aloud

than when it is done silently.¹ At the same time oral analysis in doubtful cases is helpful, and in this respect examination of the pupil's methods is made simpler and more effective if we are conversant with the common errors in the four processes. For example if we know that in column addition, as well as the obvious mistakes such as errors in combinations ($39 + 7 = 45$), failure to carry, or omission of a number, there are other less obvious mistakes such as adding in the carrying number irregularly—sometimes first, sometimes last, sometimes in the middle of the column—or useless splitting of numbers, e.g. $49 + 6 = 49 + 1 = 50 + 5 = 55$, then we have prior knowledge that makes our oral examination of the scholar's work both keener and more effective. Hence schedules of common errors in the four rules, in order of importance, together with examples of these, are a further means by which the diagnosis of pupils' difficulties can be made with both speed and accuracy.

We now turn to a more detailed discussion of specific diagnostic tests in arithmetic, namely the Schonell Diagnostic Arithmetic Tests,² and to the examination of schedules of common errors compiled from experience with them.

¹ The fact is not neglected that there are also a few pupils of the reverse type who work sums orally but are inaccurate in written arithmetic.

² These are published in a sixteen-page booklet by Oliver & Boyd, Ltd., Edinburgh and London. It will be found helpful if a copy of the test is used for reference in conjunction with the reading of Chapter II.

CHAPTER II

THE SCHONELL DIAGNOSTIC ARITHMETIC TESTS : THEIR NATURE AND ADMINISTRATION

THE Schonell Diagnostic Arithmetic Tests consist of twelve tests constructed for the purpose of gauging levels of attainment and for locating individual difficulties in addition, subtraction, multiplication, division (short and long) and simple mental problems. The tests are as follows :

- Test 1. Addition (100 basic number combinations).
- Test 2. Subtraction (100 basic number combinations).
- Test 3. Multiplication (100 basic number combinations).
- Test 4. Division (90 basic number combinations).
- Test 5. Miscellaneous. (Difficult number combinations in the four processes together with the more difficult examples in multiplication and division by 10, 11, 12.)
- Test 6. Graded Addition.
- Test 7. Graded Subtraction.
- Test 8, A and B. Graded Multiplication.
- Test 9. Graded Division.
- Test 10. Graded Long Division. (Easy steps.)
- Test 11. Graded Long Division. (Harder steps.)
- Test 12. Graded Mental Arithmetic.

NATURE OF EACH TEST

In the following pages the nature of each test is dealt with in detail, content, construction and values being considered.

TEST I. ADDITION

This test consists of the 100 basic addition facts which cover all combinations of numbers under 10, inclusive of zero combinations. There is an approximate progression of difficulty throughout the series, but no serious attempt has been made in this direction, for modern research shows that degree of difficulty in the various number combinations in the four processes differs considerably from pupil to pupil—a combination that is easy for one pupil to master is difficult for another and vice versa—a point which indicates the paramount importance of tracing the difficulties of each individual. Combinations are also given in two forms, for it is found that not infrequently a pupil who knows $9 + 7 = 16$ may fail with $7 + 9$. The material is arranged in groups of five combinations across the page and in twenty columns down the page with letters across and down to indicate exact positions of lines and items when the test is being given or corrected orally. The first and last three lines of the test are reproduced below :

TEST I. ADDITION

These are all "ADD" sums.

Work across the page.

	(a)	(b)	(c)	(d)	(e)
A.	$1 + 1 =$	$0 + 0 =$	$2 + 2 =$	$2 + 1 =$	$1 + 3 =$
B.	$2 + 0 =$	$3 + 1 =$	$3 + 3 =$	$5 + 5 =$	$4 + 1 =$
C.	$1 + 6 =$	$4 + 0 =$	$4 + 4 =$	$1 + 7 =$	$6 + 1 =$
R.	$7 + 5 =$	$5 + 9 =$	$4 + 9 =$	$8 + 6 =$	$7 + 8 =$
S.	$9 + 5 =$	$8 + 7 =$	$6 + 9 =$	$9 + 8 =$	$9 + 7 =$
T.	$6 + 8 =$	$9 + 6 =$	$8 + 5 =$	$5 + 8 =$	$7 + 9 =$

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The value of the test lies in the fact that, in so far as it includes all basic combinations, it is a means of systematically assessing the pupil's attainments in simple addition. It shows whether any particular combination, through accident or wrong association, has remained unknown or uncertain. To some extent the teacher would discover, during ordinary arithmetic lessons, a number of the deficiencies that the test reveals in his pupils, but it is also clear from experience that large numbers of individual errors remain undetected. For example, use of the test with a girl aged 11, very backward in arithmetic, showed that, amongst other errors, she always made the mistake $9 + 6 = 14$ and had been consistently inaccurate in this combination for months past.

Naturally the test is not free from the possibility of chance inaccuracy ; that is from slips, through speed or slackening of concentration, that pupils normally make, but these are few. Where a combination is wrong in one form only the inaccuracy may be due to chance or to limited knowledge, but where it is wrong in both forms (e.g. $6 + 7 = 11$, $7 + 6 = 11$) the teacher can be fairly certain that the combination is not known by the child. Furthermore, as with tests of basic combinations in the other processes, so in addition, one can verify the nature of the error by testing the combination again orally. A major value of the test is that it determines how far pupils, particularly younger and more backward ones, have proceeded with abstract addition. If the number of errors in the test is very great it is quite clear that the pupil is not ready to proceed with addition "sums" ; he has not successfully bridged that gulf from the concrete to the abstract, and it is waste of the teacher's time to require him to add 26 and 83 when he has not mastered the basic combinations involved. He requires to be turned back to simpler

material with concrete aids, or alternatively he should use concrete counting aids while doing his simple addition sums.

HIGHER DECADE ADDITION—SUPPLEMENTARY
TESTS X AND Y

A final point of importance is that concerning higher decade addition. The discerning teacher may observe that a child knows $2 + 7$, but does not correctly add $52 + 7$, or gives the correct answer to $6 + 9$, but errs in $36 + 9$; and he will be led to ask, "How far is there transfer from the simple combinations to the higher decade combinations?" In other words, if pupils know all the 100 basic combinations under 10, are they likely to succeed with the same combinations in numbers over 10? The answer is that most pupils of normal or supernormal intelligence make the necessary transfer, but dull pupils and those experiencing difficulty with arithmetic require further aid. Now, in column addition of the kind that most pupils will experience in everyday life there are 225 of these higher decade addition facts. Therefore the 100 most difficult higher decade combinations have been selected, on the basis of error frequency, and set out in *Appendix I* as SUPPLEMENTARY ADDITION TEST X.

Similarly there is a certain amount of addition in simple multiplication sums, for the higher decade addition combinations involved in multiplication will always have as their basis a multiplication fact. This is evident from a study of the following:

$$\begin{array}{r} 78,429 \\ \times 9 \\ \hline \end{array}$$

Here the pupil multiplies 9 by 9 (81), he puts down 1 and carries 8, 9×2 (18), 18 and 8 are 26, put

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down 6 and carry 2, 9×4 (36), 36 and 2 are 38, put down 8 and carry 3, 9×8 (72), 72 and 3 are 75, put down 5 and carry 7, 9×7 (63), 63 and 7 are 70.

Of the combinations such as 18 and 8, 36 and 2, 72 and 3, 63 and 7, there are in all 175, and these cover every possible addition combination in the use of multiplication tables up to 9 (inclusive). As success in simple multiplication is dependent upon correct use of these addition facts, and as there are relatively few of them, it should be the aim of all teachers to test their classes systematically with the combinations and then to provide individual drill for those pupils requiring it. To facilitate this 160 of these combinations have been selected and included in SUPPLEMENTARY ADDITION TEST Y. The 45 omitted combinations are simple ones such as 24 and 1, 10 and 2, in which there is a very small amount of error.

Test Y, which, like Test X, can be given orally by the teacher, includes also some of the most difficult combinations that occur in exercises involving 10, 11 and 12 times multiplication tables. Should the class teacher find that his class is making an undue proportion of errors in multiplication by 10, 11 and 12, then it would be advisable to provide table practice in which the numbers, 2, 3, 4, 5, 6 etc. are, in turn, added to the separate products in each of the tables.

In the test of the 100 basic combinations (Test 1) a common error is that of substituting multiplication for addition, thus $4 + 4 = 16$, $3 + 3 = 9$, $6 + 2 = 12$. The most difficult combinations are : $9 + 8$, $7 + 9$, $5 + 9$.

TEST 2. SUBTRACTION

This test, similar in pattern to Test 1, consists of the 100 basic subtraction combinations which are

fundamental, not only to subtraction itself, but to division, and to simple problems. The test enables the teacher to examine scientifically all his pupils in all subtraction combinations, not once but a number of times, and then to distribute drill according to individual needs.

The first and last three lines of the test are as follows :

TEST 2. SUBTRACTION

These are all "SUBTRACT" or "TAKE AWAY" sums.

Work across the page.

	(a)	(b)	(c)	(d)	(e)
A.	$3 - 2 =$	$1 - 1 =$	$4 - 2 =$	$5 - 4 =$	$0 - 0 =$
B.	$5 - 3 =$	$3 - 3 =$	$5 - 1 =$	$4 - 4 =$	$8 - 1 =$
C.	$6 - 6 =$	$4 - 3 =$	$3 - 1 =$	$2 - 1 =$	$6 - 5 =$
R.	$14 - 8 =$	$15 - 9 =$	$13 - 8 =$	$15 - 6 =$	$13 - 5 =$
S.	$14 - 6 =$	$13 - 4 =$	$17 - 8 =$	$16 - 9 =$	$13 - 7 =$
T.	$17 - 9 =$	$14 - 5 =$	$14 - 9 =$	$13 - 9 =$	$16 - 7 =$

The combinations have been arranged in approximate order of difficulty, but here as in other processes there is considerable variation in the difficulty presented by any one pair of units from pupil to pupil.

With dull pupils and those backward in arithmetic it is useful to notice the errors in subtraction compared with those in addition and to see how far the pupil views them as two entirely different processes and how far he sees some relationship between them. There are some pupils, backward in arithmetic, who have never been shown the relationship between such combinations as $15 - 8 = 7$, $8 + 7 = 15$, and $15 - 7 = 8$.

The combinations most frequently incorrect are : $11 - 6$, $16 - 9$, $17 - 9$, $15 - 6$, $13 - 4$, $14 - 6$, $13 - 8$, $11 - 3$.

More errors in calculation and more substitution of another process occur in this test than in addition.

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TEST 3. MULTIPLICATION

This test provides a measure of the pupil's attainments in his multiplication tables up to 9 times. But the material is not set out in the conventional form of $1 \times 2 = 2$, $2 \times 2 = 4$, $3 \times 2 = 6$ etc., for there are not a few pupils who have so mechanically memorised the multiplication facts that when they require a certain item, say 6×8 , they have to run through in their minds all the preceding facts in the table before they can give a response. On the contrary the test contains the multiplication combinations set out in jumbled form. To facilitate testing in multiplication of 7-year-old pupils in infant classes, where the aim is to teach up to the 6 times tables, all combinations up to 9×6 have been grouped in the first 11 lines (A-K) of the test. A line separates these from the remainder of the test, but naturally most pupils would do the entire series. Combinations have been used in two forms, e.g. $6 \times 7 =$ and $7 \times 6 =$, while all possible "0" difficulties, e.g. 0×0 , and 9×0 and 0×7 , have been included.

The first and last three lines of the test are reproduced below :

TEST 3. MULTIPLICATION

These are all "MULTIPLY" or "TIMES" sums.
Work across the page.

	(a)	(b)	(c)	(d)	(e)
A.	$1 \times 3 =$	$2 \times 2 =$	$1 \times 7 =$	$2 \times 1 =$	$1 \times 4 =$
B.	$5 \times 1 =$	$2 \times 5 =$	$1 \times 6 =$	$2 \times 8 =$	$1 \times 5 =$
C.	$4 \times 1 =$	$2 \times 3 =$	$1 \times 8 =$	$3 \times 2 =$	$2 \times 9 =$
R.	$8 \times 7 =$	$7 \times 9 =$	$3 \times 0 =$	$9 \times 6 =$	$7 \times 7 =$
S.	$8 \times 0 =$	$6 \times 0 =$	$7 \times 8 =$	$8 \times 5 =$	$0 \times 2 =$
T.	$9 \times 7 =$	$9 \times 3 =$	$0 \times 1 =$	$0 \times 7 =$	$9 \times 4 =$

Use of the test reveals the elements in the multiplication tables that have been insufficiently memorised. It also emphasises the need to teach all

combinations in both forms ; the pupil who gets 4×9 right but 9×4 wrong has not only learnt these as unrelated items but has, in all probability, given insufficient attention to 9×4 because it was considered that 4×9 in the 4 times table would transfer to the item in reverse form in the 9 times table. Such an assumption is, in the main, not justified by results of this test.

The most difficult combinations are : 9×6 , 7×8 , 9×7 and all the zero combinations.

TEST 4. DIVISION

Test 4 provides a comprehensive survey of the 90 basic division facts. As in Test 3 they are arranged so that lines A-K include division by numbers from 1 to 6, while lines L-S cover division by 7, 8 and 9 and the 0 difficulties. This arrangement facilitates the testing of pupils in upper classes of infant departments.

The first and last three lines of the test are as follows :

TEST 4. DIVISION

These are all "DIVIDE" sums.

Work across the page.

	(a)	(b)	(c)	(d)	(e)
A.	$4 \div 2 =$	$10 \div 2 =$	$9 \div 3 =$	$6 \div 2 =$	$10 \div 5 =$
B.	$15 \div 3 =$	$8 \div 2 =$	$14 \div 2 =$	$20 \div 5 =$	$25 \div 5 =$
C.	$12 \div 2 =$	$16 \div 2 =$	$12 \div 6 =$	$12 \div 3 =$	$15 \div 5 =$
P.	$0 \div 2 =$	$42 \div 7 =$	$0 \div 5 =$	$36 \div 9 =$	$0 \div 7 =$
Q.	$48 \div 8 =$	$63 \div 7 =$	$0 \div 8 =$	$64 \div 8 =$	$63 \div 9 =$
R.	$0 \div 9 =$	$54 \div 9 =$	$7 \div 7 =$	$56 \div 8 =$	$9 \div 9 =$

As with the multiplication combinations, so in division, facts involving 0's cause the greatest number of errors, while $54 \div 9$, $54 \div 6$, $42 \div 7$, $24 \div 3$, $2 \div 2$, $3 \div 3$, $4 \div 4$, $5 \div 5$, $6 \div 6$, $7 \div 7$, $8 \div 8$, $9 \div 9$ also occasion difficulty.

The 90 combinations which form Test 4 are

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fundamental to all arithmetic progress and should be rendered automatic by all pupils. The facts are, however, without remainders so that there still exist 360 simple division facts that have remainders. Most of these are known if pupils master the basic facts, but attention might profitably be drawn to 105 of the most difficult ones. I have, therefore, provided in SUPPLEMENTARY TEST 2 an inventory of these further division facts with remainders that the teacher can use for oral testing with his pupils.

COMPARISON OF RESULTS IN TESTS 1 TO 4

As well as revealing individual errors in the basic combinations in the four processes, Tests 1 to 4 provide evidence on the relative degrees to which pupils have mastered the fundamental facts in addition, subtraction, multiplication and division. This important information is soon shown by a tabulation of results. For example, amongst the tabulated scores of 93 nine-year-old pupils in a junior school the following variations in accuracy were to be noted :

Possible Scores :	Addn. 100	Subn. 100	Multn. 100	Divn. 90
Pupils :—E. B. . .	97	98	97	74
J. P. . .	99	92	88	86
L. B. . .	99	99	90	52
R. S. . .	100	88	82	86
M. P. . .	97	99	96	67
R. V. . .	96	83	96	71
S. L. . .	100	90	85	84
F. P. . .	100	78	73	90

It is obvious that different pupils require practice in different fundamentals to bring them up to a normal

level of efficiency. Thus we note that L. B., although up to average in three processes, is particularly weak in division, whereas F. P. makes maximum scores in addition and division, but shows definite weakness in subtraction and multiplication.

Not only does such a comparison show deficiencies in particular processes, but it throws into relief the general weakness of some pupils. For example, in the group mentioned above there were :

J. L.	.	95	67	65	38
D. W.	:	94	71	70	62

Yet these pupils were endeavouring to work sums involving hundreds, carrying and "borrowing."

It cannot be too strongly urged that every teacher should consider, in tabulated form, the results of all his pupils in Tests 1 to 4. Time thus spent will be amply repaid in later work in arithmetic.

TEST 5. MISCELLANEOUS

The first fourteen lines of this test consist of seventy of the most difficult combinations in the four processes, as contained in Tests 1 to 4. The combinations are arranged in mixed order and are primarily for discovering the pupil's efficiency in changing from one process to another.

Examination of errors in arithmetic shows that certain pupils are unable to keep before them the single idea necessary for continuing with the same kind of response when dealing with groups of figures. Thus, instead of continuing to add in a particular set of exercises, they will change to multiplication or subtraction. Usually the direction of their ideas is changed by a combination of figures which has a stronger appeal in some process other than the one being used. In reverse manner some pupils experience difficulty in changing from one process to

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another. For example, after two additions they will continue to add where a multiplication sign indicates a change of operation. Test 5 shows the degree of facility pupils possess in responding to the more difficult combinations in the four rules when the process changes frequently.

The last thirty elements in the test consist of a selection of the most difficult combinations from the multiplication and division tables of 10, 11 and 12.

The first and last three lines of the test are shown below :

TEST 5. MISCELLANEOUS

There are FOUR kinds of sums here, "ADD," "SUBTRACT," "MULTIPLY," "DIVIDE."

Work across the page.

A.	(a) $3 + 8 =$	(b) $12 - 5 =$	(c) $7 \times 6 =$	(d) $3 + 9 =$	(e) $42 \div 7 =$
B.	$27 \div 3 =$	$5 \times 0 =$	$12 - 7 =$	$11 - 4 =$	$15 - 7 =$
C.	$9 + 3 =$	$36 \div 9 =$	$7 \times 9 =$	$7 + 6 =$	$8 + 9 =$
R.
S.	$72 \div 12 =$	$11 \times 10 =$	$11 \times 8 =$	$132 \div 12 =$	$12 \times 11 =$
T.	$11 \times 9 =$	$77 \div 11 =$	$144 \div 12 =$	$12 \times 12 =$	$110 \div 11 =$
	$48 \div 12 =$	$12 \times 5 =$	$99 \div 11 =$	$132 \div 11 =$	$12 \times 9 =$

It is sometimes useful, particularly with older pupils, to discover their levels in speed and accuracy with the basic materials provided in Tests 1 to 5. On page 21, therefore, I have set out figures showing the average achievements to be expected in each of these tests from pupils in each of the age groups 7 to 14 years when time limits are set. Using the tests in this way the teacher should observe the following time limits :

Test 1, 3 minutes ; Test 2, $3\frac{1}{2}$ minutes ;
Test 3, 3 minutes ; Test 4, $3\frac{1}{2}$ minutes ;
Test 5, 5 minutes.

The table below gives the average scores made by each of the age groups 7 to 14 years (2400 pupils).

TABLE A. AVERAGE SCORES IN GIVEN TIMES FOR TESTS 1 TO 5 FOR AGE GROUPS 7 TO 14 YEARS

	Ages in Years							
	7	8	9	10	11	12	13	14
Test 1 (in 3 mins.)	32	43	57	70	80 (73)	87 (81)	93 (87)	96 (92)
Test 2 (in 3½ mins.)	30	41	57	70	80 (72)	86 (76)	90 (80)	94 (84)
Test 3 (in 3 mins.)	21	31	44	58	70 (59)	78 (65)	84 (70)	87 (76)
Test 4 (in 3½ mins.)	13	21	34	50	64 (53)	74 (59)	80 (65)	84 (74)
Test 5 (in 5 mins.)	8	16	30	42	56 (44)	68 (51)	76 (57)	81 (65)

The scores were derived from normal samples of the school population between the ages of 7 and 14+. The figures in brackets, for ages 11 to 14, apply only to pupils in those types of post-primary schools already "creamed" of pupils who have gone to secondary or other forms of higher post-primary schools.

All pupils actually wrote their answers on the test paper. It is apparent, therefore, that, if a pupil's scores are to be compared with the average scores given, the test must be conducted under similar conditions, that is, Tests 1 to 5 cannot be taken orally, nor can the answers be written on a previously prepared answer sheet (see page 38). The answer must be written in the test booklet if a comparative estimate of combined speed and accuracy is required.

This use of the test booklet does not preclude a full diagnostic value being obtained; for, after the tests have been marked, the teacher can still allow pupils to complete each of the tests in their own time and thus discover any individual difficulties.

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TEST 6. GRADED ADDITION

In Test 6 a complete inventory of all reasonable steps in ordinary addition has been provided. The steps have been graded in order of difficulty and four examples of the same type have been allotted to each step. Preliminary trials with this type of diagnostic test show that four examples are sufficient to provide reliable evidence regarding the pupil's degree of mastery of succeeding phases in the four rules. If a pupil fails in all examples of a particular step or gets only one correct, then it can be safely assumed that he is unfamiliar with the actual nature of the step. Of course, it might be that he is grossly inaccurate in the basic combinations involved in the step, but if this is a general condition, which can be determined from the results of Tests 1 to 5, then it amounts to the same thing as not knowing the step, for it means that the pupil's basic knowledge in addition, subtraction, multiplication or division is so uncertain that he requires considerably more experience with the concrete and with drills before he should be allowed to try the more difficult phases of a process. If, on the other hand, a pupil succeeds with three or four of the examples allotted to a step, it is safe to assume that he is at least conversant with the aspect of the process that the step represents.

In the test booklet there are eight examples to a line, i.e. two testing units ; for example, the first two lines are as follows :

TEST 6. GRADED ADDITION

These are all "ADD" sums.
Work across the page.

A.	(a) 14 3 —	(b) 15 4 —	(c) 12 6 —	(d) 2 17 —	(e) 10 15 —	(f) 13 16 —	(g) 12 14 —	(h) 11 10 —
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B.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
	31	65	23	28	123	346	482	543
	66	22	73	30	45	212	305	126
	—	—	—	—	—	—	—	—

These examples represent four steps in the addition test. Below, each unit of four examples is set out separately, with a description of the step it is designed to test. This arrangement will facilitate later interpretation of results and distribution of remedial work.

1st step :

14	15	12	2	Tens (under 20) in one line, units in the other ; no carrying.
3	4	6	17	—
—	—	—	—	—

2nd step :

10	13	12	11	Tens (under 20) in both lines ; 0's introduced ; no carrying.
15	16	14	10	—
—	—	—	—	—

3rd step :

31	65	23	28	Tens (over 20) in both lines ; no carrying.
66	22	73	30	—
—	—	—	—	—

4th step :

123	346	482	543	Hundreds and tens in both lines ; no carrying.
45	212	305	126	—
—	—	—	—	—

5th step :

9	15	6	9	Units in one line, tens (under 20) in the other ; carrying.
19	6	17	17	—
—	—	—	—	—

6th step :

57	58	6	7	Units in one line, tens (over 20) in the other ; carrying.
7	6	89	68	—
—	—	—	—	—

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7th step :

$$\begin{array}{r} 23 \\ 17 \\ \hline \end{array} \quad \begin{array}{r} 39 \\ 48 \\ \hline \end{array} \quad \begin{array}{r} 14 \\ 79 \\ \hline \end{array} \quad \begin{array}{r} 37 \\ 59 \\ \hline \end{array} \quad \text{Tens in both lines, carrying in units place.}$$

8th step :

$$\begin{array}{r} 87 \\ 31 \\ \hline \end{array} \quad \begin{array}{r} 96 \\ 63 \\ \hline \end{array} \quad \begin{array}{r} 84 \\ 94 \\ \hline \end{array} \quad \begin{array}{r} 50 \\ 81 \\ \hline \end{array} \quad \text{Tens in both lines, carrying in tens place.}$$

9th step :

$$\begin{array}{r} 209 \\ 39 \\ \hline \end{array} \quad \begin{array}{r} 874 \\ 83 \\ \hline \end{array} \quad \begin{array}{r} 635 \\ 944 \\ \hline \end{array} \quad \begin{array}{r} 401 \\ 607 \\ \hline \end{array} \quad \text{Numbers over 100 in one or both lines; carrying in units, tens or hundreds place.}$$

10th step :

$$\begin{array}{r} 56 \\ 69 \\ \hline \end{array} \quad \begin{array}{r} 38 \\ 86 \\ \hline \end{array} \quad \begin{array}{r} 57 \\ 59 \\ \hline \end{array} \quad \begin{array}{r} 54 \\ 97 \\ \hline \end{array} \quad \text{Tens in both lines; carrying in both units and tens places.}$$

11th step :

$$\begin{array}{r} 74 \\ 56 \\ 43 \\ \hline \end{array} \quad \begin{array}{r} 38 \\ 78 \\ 94 \\ \hline \end{array} \quad \begin{array}{r} 46 \\ 37 \\ 96 \\ \hline \end{array} \quad \begin{array}{r} 86 \\ 48 \\ 39 \\ \hline \end{array} \quad \text{Column addition, 3 lines; numbers under 100; carrying.}$$

12th step :

$$\begin{array}{r} 897 \\ 497 \\ \hline \end{array} \quad \begin{array}{r} 953 \\ 818 \\ \hline \end{array} \quad \begin{array}{r} 765 \\ 488 \\ \hline \end{array} \quad \begin{array}{r} 925 \\ 469 \\ \hline \end{array} \quad \text{Hundreds, tens and units in both lines; carrying in 2 or 3 places.}$$

13th step :

$$\begin{array}{r} 77 \\ 48 \\ 32 \\ 65 \\ \hline \end{array} \quad \begin{array}{r} 94 \\ 83 \\ 76 \\ 59 \\ \hline \end{array} \quad \begin{array}{r} 277 \\ 183 \\ 149 \\ \hline \end{array} \quad \begin{array}{r} 126 \\ 848 \\ 976 \\ \hline \end{array} \quad \text{Column addition, 4 lines of 2 figures; 3 lines of 3 figures.}$$

14th step :

$$\begin{array}{r}
 28 & 608 & 3 & 951 \\
 103 & 705 & 81 & 382 \\
 784 & 33 & 19 & 467 \\
 9 & 219 & 827 & 539 \\
 \hline
 \hline
 \end{array}$$

Variations in column addition introducing difficult number combinations.

At the conclusion of the test there are two examples of addition in a horizontal setting :

$$6 + 4 + 9 + 7 + 8 + 5 + 3 + 9$$

$$9 + 8 + 7 + 6 + 5 + 8 + 7 + 3 + 2 + 0 + 7$$

With the aid of the above analysis the teacher can examine the results of his pupils, particularly backward ones, and find the exact level that they have reached in addition. The field of possible error is covered step by step so that with all pupils, irrespective of age or arithmetical attainments, the teacher is enabled to detect weak spots in this process. This systematic review of all possible difficulties provides the teacher not only with the necessary information for future class-work in the addition process, but also makes it possible to give the exact aid required by individual pupils.

All types of errors are exposed by the test. For example it was clear from the test that Ruth S., aged $8\frac{1}{2}$, had never learnt to carry. She worked the first sixteen sums correctly, but after that all her efforts were of this type :

$$\begin{array}{cccc}
 9 & 15 & 6 & 9 \\
 19 & 6 & 17 & 17 \\
 \hline
 118 & 111 & 113 & 116 \\
 \\
 57 & 58 & 6 & 7 \\
 7 & 6 & 89 & 68 \\
 \hline
 514 & 514 & 815 & 615
 \end{array}$$

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In other cases the results show evidence of persistent errors in certain combinations; thus Eileen B., aged $9\frac{5}{12}$, makes these mistakes:

$$\begin{array}{r} 96 \\ 63 \\ \hline 129 \end{array} \quad \begin{array}{r} 635 \\ 944 \\ \hline 1279 \end{array} \quad \begin{array}{r} 56 \\ 69 \\ \hline 122 \end{array}$$

For this pupil $9 + 6 = 12$.

Types of carrying errors are also plainly revealed; whereas some pupils will correctly carry figures over 1 they not infrequently make a mistake when the carrying figure is 1. These and other types of error are discussed, from the point of view of frequency and importance, in a later section.

TEST 7. GRADED SUBTRACTION

The principle in this test is similar to that of Test 6. The aim is to cover by successive steps of increasing difficulty all the major steps in the process of subtraction. Four examples are selected to test each step, and two of these units of four, that is eight examples, appear in each of the seven lines of the test booklet. To explain the test fully and to assist the teacher in examination of results, each step is here considered separately:

1st step :

$$\begin{array}{rrrr} 98 & 57 & 84 & 38 \\ 3 & 4 & 1 & 8 \\ \hline \underline{-} & \underline{-} & \underline{-} & \underline{-} \end{array} \quad \text{Tens and units in minuend; units in subtrahend; no borrowing.}$$

2nd step :

$$\begin{array}{rrrr} 55 & 99 & 78 & 97 \\ 32 & 43 & 10 & 22 \\ \hline \underline{-} & \underline{-} & \underline{-} & \underline{-} \end{array} \quad \text{Tens and units in minuend and subtrahend; no borrowing.}$$

3rd step :

346 987 378 496 Hundreds, tens and units minuend
215 832 122 261 and subtrahend; no borrowing.

4th step :

18 19 16 17 Numbers less than 20. Unit digit in
14 18 10 15 subtrahend less than unit digit in
 minuend; tens digits both unity.

5th step :

71 62 46 84 Tens in minuend, units in subtra-
2 4 7 6 hend; borrowing.

6th step :

54 22 58 46 Tens and units in minuend and
39 17 19 27 subtrahend; borrowing in units.

7th step :

331 543 283 786 Hundreds, tens and units in minuend;
18 25 29 58 borrowing in units.

8th step :

316 564 68 387 Borrowing in units and tens or bor-
27 59 59 299 rowing in units and zero result in
 tens.

9th step :

80 980 430 168 Introduction of zero difficulty in
57 930 416 68 units or tens.

10th step :

180 250 160 890 Examples of 0 and 0 difficulties.
71 49 31 889 1 9

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11th step :

$$\begin{array}{cccc} 346 & 629 & 756 & 387 \\ \underline{284} & \underline{473} & \underline{382} & \underline{196} \end{array} \quad \text{Borrowing in tens place. . Numbers over 100.}$$

12th step :

$$\begin{array}{cccc} 364 & 831 & 8354 & 8112 \\ \underline{295} & \underline{276} & \underline{5676} & \underline{6798} \end{array} \quad \text{Borrowing in hundreds, tens and units places.}$$

13th step :

$$\begin{array}{cccc} 800 & 607 & 700 & 906 \\ \underline{695} & \underline{298} & \underline{192} & \underline{199} \end{array} \quad \text{Advanced "0" difficulties and borrowing.}$$

14th step :

$$\begin{array}{cccc} 891 & 904 & 705 & 6067 \\ \underline{207} & \underline{206} & \underline{109} & \underline{5970} \end{array} \quad \text{Advanced "0" difficulties and borrowing.}$$

The above examination of the test makes it apparent that any difficulty in subtraction would be revealed by use of the test.

Particularly does the test show the nature of carrying errors prevalent amongst children, and the widespread difficulties occasioned by like numbers and by noughts in subtraction. These latter points are clearly portrayed in the following results of *Kenneth R.*, age 10 :

1st step. One error.

$$\begin{array}{r} 38 \\ - 8 \\ \hline 38 \end{array}$$

2nd and 3rd steps. No errors.

4th step. Four errors.

$$\begin{array}{rrrr} 18 & 19 & 16 & 17 \\ - 14 & - 18 & - 10 & - 15 \\ \hline 14 & 11 & 16 & 12 \end{array}$$

5th, 6th, 7th and 8th steps. No errors.

9th step. Two errors.

80	430
57	416
30	426

10th step. Four errors.

180	250	160	890
71	49	31	889
111	219	131	819

11th and 12th steps. No errors.

13th step. Four errors.

800	607	700	906
695	298	192	199
295	509	692	907

14th step. Three errors.

904	705	6067
206	109	5970
708	616	1917

The nature of Kenneth's errors with like numbers and with noughts, both in consistency and extent, is amply illustrated.

The individual form of difficulties in arithmetic and the illogicality of their causal basis is only clearly revealed by use of diagnostic tests ; thus Kenneth can correctly work the apparently difficult examples in the 12th step :

8354	and	8112
<u>5676</u>		<u>6798</u>
—		—

because they do not contain like figures or "0" difficulties, but he fails with simple sums like

18	160	180
<u>14</u>	<u>31</u>	<u>71</u>
—		—

TEST 8, A AND B. GRADED MULTIPLICATION

This test provides a gauge of attainment in multiplication simple and compound. From a very

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simple example (22×4) the test proceeds by steps of increasing difficulty to exercises in three figure multiplication (6328×179). As in Tests 6 and 7 there are four items of similar type to each step, but, in order to allow adequate space for working items in the test booklet, only six examples are allotted to each line on the first page of the test and four on the second page. Thus on the first page of the test each line contains four examples of one step and two of the succeeding step, which is completed on the next line with two more examples. The first two lines of the test, containing twelve examples, i.e. three steps, are as follows :

A.	(a) 22 4	(b) 31 5	(c) 63 2	(d) 91 5	(e) 423 3	(f) 612 4
	—	—	—	—	—	—
B.	711 5	843 2	60 6	303 3	400 8	9010 5
	—	—	—	—	—	—

There are in the complete test fourteen steps embracing fifty-three examples. These, with descriptive details, are set out below :

1st step :

22 — 4	31 — 5	63 — 2	91 — 5	Simple multiplication, 2 figures in multiplicand ; no carrying.
—	—	—	—	—

2nd step :

423 — 3	612 — 4	711 — 5	843 — 2	Simple multiplication, 3 figures in multiplicand ; no carrying.
—	—	—	—	—

3rd step :

60 — 6	303 — 3	400 — 8	9010 — 5	Simple multiplication, 2, 3 or 4 figures in multiplicand. "0" difficulties introduced ; no carrying.
—	—	—	—	—

4th step :

$$\begin{array}{r} 18 \\ \times 9 \\ \hline 162 \end{array}$$

$$\begin{array}{r} 19 \\ \times 7 \\ \hline 133 \end{array}$$

Simple multiplication ; carrying into tens place.

5th step :

$$\begin{array}{r} 76 \\ \times 9 \\ \hline 684 \end{array}$$

$$\begin{array}{r} 96 \\ \times 6 \\ \hline 576 \end{array}$$

Simple multiplication, multiplicand over 20 ; carrying into tens place.

6th step :

$$\begin{array}{r} 104 \\ \times 9 \\ \hline 936 \end{array}$$

$$\begin{array}{r} 106 \\ \times 11 \\ \hline 1166 \end{array}$$

Simple multiplication, numbers over 100. "0" difficulties ; carrying.

7th step :

$$\begin{array}{r} 348 \\ \times 12 \\ \hline 696 \end{array}$$

$$\begin{array}{r} 4196 \\ \times 11 \\ \hline 46156 \end{array}$$

Simple multiplication, 3 to 6 figures in multiplicand ; carrying.

8th step :

$$\begin{array}{r} 34 \\ \times 22 \\ \hline 68 \end{array}$$

$$\begin{array}{r} 52 \\ \times 31 \\ \hline 164 \end{array}$$

Compound multiplication, 2 figures in multiplicand and multiplier. Two examples ; no carrying. Two examples ; carrying.

9th step :

$$\begin{array}{r} 80 \\ \times 97 \\ \hline 740 \end{array}$$

$$\begin{array}{r} 60 \\ \times 84 \\ \hline 504 \end{array}$$

Compound multiplication, 2 figures in multiplicand and multiplier. "0" difficulties ; carrying.

10th step :

$$\begin{array}{r} 90 \\ \times 90 \\ \hline 8100 \end{array}$$

$$\begin{array}{r} 8460 \\ \times 600 \\ \hline 507600 \end{array}$$

Extension of "0" difficulties.

11th step :

$$\begin{array}{r} 483 \\ \times 59 \\ \hline 4510 \end{array}$$

$$\begin{array}{r} 976 \\ \times 75 \\ \hline 78 \end{array}$$

Compound multiplication, 3 figures in multiplicand ; carrying.

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12th step :

$$\begin{array}{r} 605 \\ 29 \end{array} \quad \begin{array}{r} 408 \\ 37 \end{array} \quad \begin{array}{r} 206 \\ 50 \end{array} \quad \begin{array}{r} 607 \\ 60 \end{array} \quad \text{Compound multiplication, 3 figures in multiplicand, tens figure "0"; carrying.}$$

13th step :

$$\begin{array}{r} 976 \\ 78 \end{array} \quad \begin{array}{r} 612 \\ 517 \end{array} \quad \begin{array}{r} 706 \\ 309 \end{array} \quad \begin{array}{r} 338 \\ 430 \end{array} \quad \text{3 figures in multiplicand and in multiplier.}$$

14th step :

$$\begin{array}{r} 7651 \\ 301 \end{array} \quad \begin{array}{r} 7080 \\ 605 \end{array} \quad \text{4 figures in multiplicand, 3 figures in multiplier.}$$

The test takes longer than preceding ones and should be taken in two parts; the examples on page 9 (Test 8A) of the test booklet at one testing and those on page 10 (Test 8B) at another testing.

TEST 9. GRADED DIVISION

This test consists of eleven steps of four examples each, i.e. forty-four simple division sums in all, involving the use of divisors from 2 to 12. Adequate attention has been paid to the main zero difficulties that occur in simple division. Examples are arranged four in a line, i.e. one step per line.

1st step :

$$\begin{array}{r} 4\overline{)44} \\ 2\overline{)84} \\ 3\overline{)96} \\ 6\overline{)666} \end{array} \quad \text{Divisor is contained an even number of times in every figure of the dividend; no carrying; no remainders.}$$

2nd step :

$$\begin{array}{r} 2\overline{)682} \\ 4\overline{)844} \\ 3\overline{)696} \\ 2\overline{)2426} \end{array} \quad \text{Same as step 1, but with larger numbers; no carrying; no remainders.}$$

3rd step :

3) <u>906</u>	2) <u>806</u>	4) <u>840</u>	3) <u>690</u>	" 0 " at end or in middle of dividend ; no carrying ; no remainders.
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4th step :

4) <u>800</u>	3) <u>900</u>	6) <u>600</u>	8) <u>16400</u>	Double " 0 " at end of dividend ; noughts in quotient ; no carrying ; no remainders.
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5th step :

5) <u>1515</u>	7) <u>6342</u>	9) <u>8136</u>	8) <u>4856</u>	Divisor is not contained in 3rd figure of dividend (giving nought in quotient), but it is contained in last 2 figures of dividend ; no carrying ; no remainders.
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6th step :

4) <u>27</u>	8) <u>53</u>	9) <u>80</u>	7) <u>61</u>	Dividends under 100 ; remainders.
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7th step :

7) <u>50</u>	4) <u>97</u>	9) <u>89</u>	6) <u>57</u>	Similar to step 6.
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8th step :

5) <u>156</u>	4) <u>167</u>	7) <u>149</u>	6) <u>128</u>	3-figure dividends ; no carrying ; remainders.
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9th step :

9) <u>372</u>	12) <u>759</u>	8) <u>697</u>	11) <u>569</u>	3-figure dividends ; larger divisors ; carrying ; remainders.
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10th step :

3) <u>248</u>	6) <u>745</u>	5) <u>3462</u>	7) <u>5573</u>	Larger dividends ; carrying ; remainders.
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11th step :

8)29643 5)25357 7)49010 9)307868 5 or 6 figure dividends ; "0" difficulties ; carrying ; remainders.

TEST 10. LONG DIVISION

Both Tests 10 and 11 deal with the most difficult process in elementary arithmetic, namely long division. Naturally the possible number of examples in relatively simple long division is exceedingly great, but an attempt has been made to compile two tests which will give a fairly comprehensive survey of the major long division difficulties. Four examples are allotted to each step, and each step occupies a single line in the booklet.

1st step :

20)40 33)99 43)86 21)84 The simplest step in long division ; the quotient, which consists of one figure, is apparent at sight from the nature of the figures in divisor and dividend ; no remainders.

2nd step :

23)48 32)99 24)49 20)86 This step is only & very little different from step 1 ; the quotient is apparent from the nature of the first figures of divisor and dividend, but there is a remainder.

3rd step :

$22\overline{)56}$ $31\overline{)98}$ $43\overline{)89}$ $20\overline{)97}$ This step is a slight advance on step 2, as the quotient is not so apparent in two examples.

4th step :

$33\overline{)74}$ $42\overline{)93}$ $23\overline{)72}$ $41\overline{)90}$ In this step the first figure of the divisor is not contained equally in the first figure of the dividend. There is a single figure quotient, but a double figure remainder.

5th step :

$21\overline{)126}$ $32\overline{)128}$ $41\overline{)164}$ $53\overline{)159}$ The first figure of the divisor in this step is not contained at all in the first figure of the dividend, but in the first two figures; 3-figure dividend; single figure quotient; no remainders.

6th step :

$43\overline{)139}$ $94\overline{)189}$ $71\overline{)288}$ $82\overline{)248}$ This is similar to step 5, but with remainders.

7th step :

$52\overline{)1456}$ $36\overline{)756}$ $63\overline{)1449}$ $97\overline{)3298}$ There are 4 figures in the dividend; 2 figures in the quotient; no remainders in this step.

8th step :

$41\overline{)3199}$ $84\overline{)5379}$ $95\overline{)7698}$ $93\overline{)5866}$ This is similar to step 7, but has single figure remainders.

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9th step :

48)3360 81)4050 54)2700 56)2800 This is similar to steps 7 and 8, but has a "0" in the quotient.

TEST 11. LONG DIVISION

This test is a continuation of Test 10 so that the steps follow on in approximate order of difficulty from the 9th step in Test 10. Preliminary trials showed that it was better to divide the long division material into two shorter tests, but with older pupils who are dull or very backward in arithmetic it is advisable still further to divide this test by giving lines A, B and C at the first testing and D, E and F at the second testing.

1st step :

36)723 55)1769 63)3909 73)5116 This step, while keeping two examples similar to those in step 8 of Test 10, introduces a "0" into the quotient of the two remaining examples and has single figure remainders.

2nd step :

18)54 39)156 35)210 26)104 This step introduces trial divisors in all examples. There are single figure quotients and no remainders.

3rd step :

15)115 49)366 29)261 68)615 This is an extension of step 2 in so

far as more trials are required to find the first quotient figure, and there are remainders.

step :

$$\begin{array}{r} 5 \\ \overline{)27)918} \end{array} \quad \begin{array}{r} 46 \\ \overline{)3012} \end{array} \quad \begin{array}{r} 24 \\ \overline{)984} \end{array}$$

In this step there are trial divisors, 2 figures in the quotient and no remainders.

step :

$$\begin{array}{r} 5 \\ \overline{)36)13657} \end{array} \quad \begin{array}{r} 78 \\ \overline{)14742} \end{array} \quad \begin{array}{r} 29 \\ \overline{)17632} \end{array}$$

This step introduces 3-figure quotients with remainders.

step :

$$\begin{array}{r} 5 \\ \overline{)79)3090} \end{array} \quad \begin{array}{r} 21 \\ \overline{)17659} \end{array} \quad \begin{array}{r} 58 \\ \overline{)46814} \end{array}$$

This step involves noughts in the quotient and some remainders.

TEST 12. GRADED MENTAL ARITHMETIC

This test is composed of forty graded examples in arithmetic. It provides an indication of how a child can handle simple mechanical material in a practical setting. The test contains examples of combinations of the four processes and applications of these involving money, weights and measures.

GENERAL INSTRUCTIONS FOR GIVING THE TESTS

The twelve tests considered in detail in foregoing paragraphs are printed in a sixteen-page booklet with space provided for answers and for working figures where these are necessary. It is obvious that

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all the tests cannot be given on the same day, but they can be completed within a week if a testing is taken each day. The distribution of the testing should depend upon the age and abilities of the pupils; with younger pupils and with those backward in arithmetic, care should be taken to avoid fatigue and boredom. For example, with 7-year-olds it is advisable to give one of Tests 1 to 5 on alternate days, i.e. one day testing followed by one day without testing. With pupils of 8 or 9 years, two of Tests 1 to 5 might be taken on one day, reserving Test 5 for a single testing.

With young or backward testees two working periods for each of Tests 6 to 10 are advisable. With tests 8B, 10 and 11 two working periods are better for all pupils up to 11+. Black lines in the actual test booklet indicate suitable diagnostic testing units.¹

In general, a more reliable assessment of arithmetical attainments is obtained if the tests are distributed over two weeks, a very short time in comparison with the extensive information gathered on individual difficulties.

Answers and working to all tests may be put in the test booklets, in which case the teacher has a comprehensive record of each pupil's achievement in the number combinations, in the four rules and in mental arithmetic. On the other hand the answers and working to the tests may be written on papers prepared by the pupils, thereby leaving the tests intact for further use. Provision for the latter form of use has been made by means of the horizontal and vertical lettering of each test. Thus if the teacher wishes to take Test 1 in this way he asks his pupils to prepare a sheet with lettering as follows :

¹ Naturally if a time limit (see pp. 21 and 56) for a test has been set, pupils are allowed to do as many examples as they can in the given time.

	TEST 1				
	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E etc.					

He then allows the children to work the test, putting their results on their answer papers in the appropriate line and under the correct letter.¹

Tests 1 to 5 may be taken orally, the pupils writing their answers either in the booklets or on prepared sheets. Oral administration of these tests has the advantage that a better estimate is obtained of the degree to which the combinations have become automatic responses, in so far as the speed at which the test is given prevents finger counting, dot counting and such devices. On the other hand, if the child is allowed to work silently at his own pace the testing is more in keeping with the majority of the arithmetic situations in which the pupil will be placed ; furthermore it is a very easy task for the teacher to note pupils who still require considerable counting aids in all the tests.

Tests 1 to 11 may be used to obtain three somewhat different measures of the child's arithmetic achievements.

- (1) The tests may be used in a purely diagnostic sense, that is, to discover as comprehensively as possible what the pupil knows and what he does not know in the number combinations, in the four rules, in the various phases of the four rules and in mental arithmetic. *Every pupil is allowed to complete as much as he can of each test in unlimited time.* Gathering of diagnostic information in this way is the

¹ This method of using the tests cannot be adopted when speed plus accuracy estimates for Tests 1 to 5 are required (see p. 21).

primary objective for which the tests are compiled. They aim to provide a complete inventory of the child's arithmetic knowledge and hence to indicate where the gaps and deficiencies lie.

- (2) The tests may also be used to estimate speed of working in the various number combinations and four rules. The time taken can be obtained by requiring each pupil to raise his hand as he finishes the test. The time taken is recorded on the top of the test paper and is afterwards compared with the table of average times for the various chronological age groups (given in the next chapter). For example, if a pupil age $9\frac{1}{2}$ takes 7 minutes to complete Test 1 when the average time for a pupil of the same age is $4\frac{1}{2}$ minutes, we have an objective estimate of his speed in adding the basic number combinations.
- (3) A combined measure of accuracy and speed may be derived from the test by setting a time limit for the test. The time limit may be determined from the average age of the class and the time norms given in Chapter III. For example, if the average age of a class were $9\frac{3}{4}$ years the time limit to set for Test 2 would be $6\frac{1}{2}$ minutes, as shown by the table of average times for age groups 8 to 13 years. Averages of the number correct with time limits have been calculated only for Tests 1 to 5. Thus what the teacher derives from using the remaining tests as measures of accuracy plus speed are comparative estimates of ability from pupil to pupil or from class to class. It is suggested that only with groups

of "A" children should the tests be used in this form. It is from their use as diagnostic tests that their main value accrues.

SPECIAL INSTRUCTIONS FOR ADMINISTERING THE TESTS

Before pupils commence a test it is as well to see that they have all turned to the particular test to be worked : test number and page number both help in this respect. Furthermore, it is advisable to impress on pupils, particularly young ones, the nature of the process they are about to do. For example, in Test 1 we should say : "Now you are going to work a lot of little addition—add—sums." (Put the sign on the blackboard.) "You will have plenty of time, but you must work quickly and carefully." Some such similar instructions should precede each test.

Care should be taken to point out that Test 5 involves four processes.

If times are being obtained for each pupil say : "Put up your hand as soon as you finish the test. Don't wait to look over it ; put your hand up."

It is as well, with younger pupils, to allow all to complete a test before proceeding with the next. Timing can be carried out with a watch having a second hand ; time to the nearest minute is sufficient.

MARKING AND SCORING THE TESTS

With young pupils aged 7 or 8, it is advisable for the teacher to correct the tests, but with older pupils it is beneficial that they should correct their own results, in which case the teacher will read the answers (given in Appendix 2) aloud slowly. Incorrect exercises should be so marked to facilitate later qualitative analysis by the teacher.

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The scoring is one mark for each correct answer. The table below gives the maximum number of marks for each test.

Test	Kind	Maximum Score
1	Addition . .	100
2	Subtraction . .	100
3	Multiplication . .	100
4	Division . .	90
5	Miscellaneous (1-4) . .	100
6	Addition . .	58
7	Subtraction . .	56
8, A & B	Multiplication . .	53
9	Division . .	44
10	Long Division . .	36
11	Long Division . .	24
12	Mental . .	40
Total maximum score		<u>801</u>

It proves useful to record results together with any special remarks regarding the pupil's work or remedial teaching on the front page of the booklet. If the pupil is very backward in arithmetic and the tests have been used in their full diagnostic sense, both as regards accuracy and time, the results might be recorded on an additional sheet, pasted inside the test, in the form shown below.

Test	Time taken	Comparison with Norms	No. Right	Comparison with Norms	Special Difficulties	Suggestions for Remedial Work
1						
2						
3						
4						
etc.						

SUMMARY OF THE USES OF THE TESTS

The detailed discussion in the foregoing pages clearly shows that the tests are primarily for diagnosing difficulties in arithmetic. They provide the class teacher with an instrument that is both scientific and systematic, and hence are of paramount value, not only with pupils backward in arithmetic, but with all children who are engaged in consolidating the fundamental processes and their applications. Extensive use of the tests in different departments during the two years devoted to preliminary testing shows that they can be employed profitably for a variety of purposes.

- (a) The tests are suitable for estimating accuracy in the fundamental combinations amongst pupils in infant classes before their transfer to junior classes.
- (b) The tests will provide useful information concerning all pupils in junior schools.
- (c) The tests are invaluable as a guide to both the attainments and the difficulties of dull pupils. They indicate starting-points and curriculum objectives for the teacher.
- (d) With children backward in arithmetic the tests isolate with certainty individual difficulties and reveal the lines along which remedial teaching should proceed.
- (e) Pupils coming to a school from another area can be given the tests to ascertain where they shall be placed with regard to future arithmetic teaching.
- (f) The tests can be used as a basis for dividing pupils into arithmetic sets, where this method of organisation is used.
- (g) Groups of pupils who are to be transferred to

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post-primary school can be tested prior to their final term in the junior school and given practice in the processes where weaknesses are still displayed.

- (h) Testing of all pupils in a senior school, but particularly the more backward ones, provides much useful information for teachers and prevents further work being attempted where the fundamentals are still not sufficiently well known.

Having examined the tests in detail and considered their administration, marking and uses, we now pass to an interpretation of the results obtained from them.

CHAPTER III

INTERPRETATION OF THE RESULTS

BEFORE considering the interpretation of the results of the tests it cannot be too strongly emphasised that they are essentially for diagnostic purposes. They are constructed according to definite principles in order to detect difficulties in arithmetic amongst ordinary primary-school pupils. It is on the qualitative not the quantitative side that they yield most information. It is one aim to discover how well a pupil can do a test or, conversely, how backward he is; but it is another and more important aim to know exactly the nature of any difficulties or gaps in his work and why he is backward in certain phases of a subject. It is this latter form of information that the diagnostic test is able to provide.

Naturally all interpretation should embrace, to some extent, both qualitative and quantitative aspects; so that, although the diagnostic test emphasises qualitative examination, it makes some provision for quantitative estimates. The figures given in the tables of averages enable separate assessments for speed and for accuracy to be made.

The number of examples correctly worked by any group of children, in the various tests, and the time taken by each pupil to complete each test can be easily ascertained. The value of such data is then enhanced if individual scores and times are compared with those of other pupils of similar ages. Such a comparison gives some idea of the pupil's achievements in terms of normal standards. This quantitative interpretation is made possible by the

use of tables of averages which have been compiled on the basis of the average number of examples correct and the average times taken by each of the age groups 8 to 13 years—age being considered as age last birthday. These averages, which must only be considered as approximations, have been calculated for normal pupils aged 8 to 13 years. There are, however, a number of points to be noted before use of the tables can be considered in detail.

Firstly, no averages were calculated for the 7-year-old groups, for it is of little benefit to obtain at that stage a quantitative assessment of such pupils' arithmetical attainments. It is much more profitable to determine the exact nature of the progress they have made in the fundamental number combinations and the difficulties they have so far encountered than to discover how much they have learnt, that is, from a purely quantitative standard of accuracy and speed. For most 7-year-old pupils the most useful material is found in Tests 1 to 4 and in the first three or four steps of Tests 6, 7, 8A and 9.

Secondly, no averages for number of items correct have been calculated for Tests 1 to 5. Since there is no time limit for the tests and the material is relatively easy, there is little to discriminate the normal 8-year-old from the normal 12-year-old on actual number right. The primary purpose of the test is to reveal individual difficulties in particular number combinations or to expose a general weakness in a particular process. Naturally there are definite differences from age group to age group in the times taken to complete these tests, and hence average times for each age group are given.

Thirdly, in so far as it is for all pupils aged 8 to 13 years irrespective of type of school or level of intelligence, that the diagnostic tests are intended, the averages for times and scores have been compiled

from results obtained within several different types of schools in order to obtain a representative set of results. That is, results were obtained from normal samples of children aged 8, 9 and 10 years in junior schools and from children 11, 12 and 13 years in senior schools, with the addition at these latter ages (11+ to 13+) of a just proportion of representatives who had been transferred to selective post-primary schools. All results thus apply to samples of normally intelligent pupils in each of the six age groups; hence scores for ordinary unreorganised schools (from which children have gone to some form of selective post-primary education) and for senior schools will be somewhat lower. In general, it was found that average scores from a normal sample of 10-year-olds in a junior school were similar to those from 11-year-olds in a senior school, while 11-, 12- and 13-year-old scores in a senior school differed from those of the total sample, i.e. (senior-school pupils plus a proportion of pupils from central and secondary schools) by 3-19 marks in the various tests.¹

It will thus be of interest for teachers to compare results from senior schools or from "C" and "D" classes in non-selective central schools with the tabulated averages which represent a normal sample of the school population.

It will be noted that, as the tests are diagnostic and hence no time limits are used, the average numbers of sums correct amongst the upper-age groups do not show, in some tests, very marked differences from group to group.

The average number of sums correct in *unlimited time* together with the backwardness levels for each of the age groups 8 to 13 years are given on pp. 48-50.

¹ Actually the differences are 4-19 marks in Tests 1-5 with an average difference of 11 marks, and 3-9 marks in Tests 6-12 with an average difference of 6 marks.

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TABLE I
TEST 6. GRADED ADDITION
NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8	50	43
9	54	48
10	55	50
11	56	51
12	57	52
13	57	53

TABLE II
TEST 7. GRADED SUBTRACTION
NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8	44	31
9	48	36
10	52	42
11	54	48
12	55	50
13	55	52

TABLE III
TEST 8, A AND B. GRADED MULTIPLICATION
NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8	20	18
9	37	26
10	43	32
11	46	38
12	50	47
13	51	49

TABLE IV
TEST 9. GRADED DIVISION

NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8	21	12
9	37	28
10	39	31
11	41	36
12	43	39
13	43	41

TABLE V
TEST 10. LONG DIVISION (EASY STEPS)

NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8
9	20	12
10	27	18
11	30	21
12	34	30
13	35	32

TABLE VI
TEST 11. LONG DIVISION (HARDER STEPS)
NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8
9
10	18	12
11	20	15
12	21	17
13	22	19

TABLE VII

TEST 12. GRADED MENTAL ARITHMETIC

NUMBER OF SUMS CORRECT IN UNLIMITED TIME

Age	Average No. Correct	Weakness indicated if below
8	12	6
9	19	13
10	30	24
11	34	28
12	36	31
13	37	33

It must be emphasised that the average scores given in Tables I to VII are to be regarded only as approximate minima around which pupils of particular ages should be grouped. The scores must not, in any sense, be regarded as final objectives ; they simply represent a normal level of accuracy in fundamental processes irrespective of speed. Pupils may be slow, but this will be detected if a time limit is set or if the time for completion of the test is taken.

To facilitate the selection of pupils who show weakness in particular processes, minimum levels have been calculated from the spread of all scores. If a pupil, with unlimited time at his disposal, fails to reach the figure set down in the third column of each table, then he must be considered as weak in the particular process and should receive individual assistance.

AVERAGES OF TIMES TAKEN FOR EACH TEST

The time that each pupil took to complete each test was obtained and, from the data, averages were

calculated. Thus the figures given on pp. 51-56 represent the average times in which pupils between the ages of 8 and 13 years can be expected to complete the various tests. The figures indicate speed of working irrespective of accuracy.

TABLE VIII
TEST 1. ADDITION
(100 Basic Combinations)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	8	11
9	6	8
10	4	6
11	4	6
12	3.5	4.5
13	3	4

TABLE IX
TEST 2. SUBTRACTION
(100 Basic Combinations)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	11	17
9	8	12
10	6	8
11	5	7.5
12	4	6
13	3.5	5

TABLE X
TEST 3. MULTIPLICATION
(100 Basic Combinations)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	9	16
9	6	12
10	4	7
11	4	6
12	3.5	5
13	3	4

TABLE XI
TEST 4. DIVISION
(90 Basic Combinations)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	13	21
9	9	14
10	6	10
11	5	9
12	4	7
13	3	5

TABLE XII
TEST 5. MISCELLANEOUS
(100 Hardest Combinations in Four Rules)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	16	23
9	13	18
10	8	12
11	7	11
12	6	9
13	5	8

TABLE XIII
TEST 6. GRADED ADDITION

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	18	25
9	11	16
10	9	13
11	9	13
12	7	10
13	6	9

TABLE XIV
TEST 7. GRADED SUBTRACTION

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8	19	27
9	13	19
10	10	15
11	9	13
12	7	10
13	6	9

TABLE XV
TEST 8, A AND B. GRADED MULTIPLICATION

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8
9	30	40
10	25	33
11	24	30
12	20	26
13	15	21

TABLE XVI
TEST 9. GRADED DIVISION

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed Indicated if A.T. is greater than
8	26	35
9	14	20
10	9	13
11	8	12
12	6	9
13	5	8

TABLE XVII
TEST 10. LONG DIVISION (EASY STEPS)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed Indicated if A.T. is greater than
8	33	46
9	28	40
10	21	30
11	19	27
12	15	22
13	11	18

TABLE XVIII

TEST 11. LONG DIVISION (HARDER STEPS)

AVERAGE TIMES IN MINUTES TAKEN TO COMPLETE
THE TEST

Age	Average Time taken	Weakness in Speed indicated if A.T. is greater than
8
9
10	25	34
11	24	31
12	22	28
13	19	26

AVERAGE SCORES FOR ACCURACY IN GIVEN TIMES

Previous tables have given separate estimates of accuracy in unlimited time (Tables I-VII) and of speed, irrespective of accuracy (Tables VIII-XVIII), for the various diagnostic tests, but what is most useful to the teacher is a combined estimate of accuracy and speed. Most class teachers wish to know how the achievements of their pupils in graded tests of the four processes compare with those of other pupils of similar ages under similar conditions. This comparison can be made by using the test booklet and by observing the following times for Tests 6 to 12 :

Test 6,	6 minutes ;	Test 7,	$5\frac{1}{2}$ minutes ;
Test 8A,	7 minutes ;	Test 8B,	7 minutes ;
Test 9,	5 minutes ;	Test 10,	9 minutes ;
Test 11,	15 minutes ;	Test 12,	10 minutes.

Naturally all the tests are not given to pupils at one testing, nor is it necessary to give all tests to all pupils. For pupils of 7+ and 8+ it is sufficient to give Tests 6, 7, 8A and 9 on separate days. For

normal pupils of 9+ and over, the work can be done in 3 testing units on separate days :

- Unit 1, Tests 6, 7, 8A (18½ minutes);
- Unit 2, Tests 8B, 9, 10 (21 minutes);
- Unit 3, Tests 11 and 12 (25 minutes).

Allow a break of 2 or 3 minutes between each test, and keep strictly to the given times. Scores are : 1 mark for each sum correct—no partial credits are allowed. Average scores¹ from 2400 pupils (fully representative samples), between the ages of 7 and 15, under the given times and conditions are set out in Appendix 1, page 100.

Tests 6 to 12 can thus be used as standardised attainment tests in arithmetic to provide a basis for comparing, in arithmetical achievements, pupils of similar ages. Teachers will also find the scores useful for speeding up accuracy in the fundamentals. The test can be repeated at intervals and progress checked by comparing scores.

Mental ages in the various arithmetical processes can also be calculated from the scores on page 100. For example, Pupil A scores 44 in Test 6, i.e. mental age between 10 and 11. Interval = 5 marks, i.e. $1 \text{ mark} = \frac{1}{5} \text{ months} \therefore 3 \text{ marks} = \frac{1}{5} \times 3 = \frac{3}{5} = 7\frac{1}{5}$ months. A's mental age for addition = 10 years 7 months. From the scores, mental ages for other processes can be similarly calculated by noting the appropriate intervals.

SCHEDULES OF COMMON ERRORS

The diagnostic test will reveal exactly the amount known by each pupil in the various processes and

¹ Norms for children in Scottish schools have been provided by Dr P. Vernon in *The Scottish Educational Journal*, Feb. 9, 1940.

will indicate the point at which failure, if any, occurs. But this is only part of a full diagnostic programme, for the nature of the errors made and why they are made should be known. In this part of the diagnosis considerable help can be obtained by familiarising ourselves with the common types of errors. Hence schedules of common errors in addition, subtraction, multiplication and division have been compiled from results of the diagnostic tests, and the types of these errors, in approximate order of frequency, are listed in the sections that follow (pp. 58-63).

The teacher will find it profitable to take with his most backward pupils an oral examination of their incorrect examples, keeping in mind the schedule of errors in the process under consideration. Causes of the errors are also indicated in the schedules.

SCHEDULE A

COMMON ERRORS IN ADDITION

A. 1. *Errors in combinations :*

Example (a)	94	Error : $18 + 4 = 21$.
	83	Here the error was a temporary lapse in accuracy. Combination is really known.
	76	
	59	
	<u>311</u>	Suggested additional practice with Test X (see Appendix I).

Example (b)	57	Error : $9 + 7 = 17$.
	59	This proves to be an habitual error for this child, as revealed by oral testing and
	<u>117</u>	Needs individual practice in plenty of small sums involving $9 + 7$ and $7 + 9$. "

by the results of Test 1. Needs individual practice in plenty of small sums involving $9 + 7$ and $7 + 9$. "

A. 2. *Omitted carrying figure :*

Example (a)	39	Error : omitted to carry 1.
	48	Here pupil omits carrying
	77	number very frequently. Obviously carrying is not an automatic process. Needs practice from beginning with simple "carrying" examples.

3	Error : omitted to carry 2.
81	Here error due to fact that
19	pupil adds carrying number
827	in whenever it makes a com-
94	plete 10, therefore it is some-
<u>1004</u>	times forgotten.

1 number :

94	Error : carried 1 instead of 2.
83	More exercises needed in col-
76	umn addition with a variety
59	of carrying numbers.
<u>302</u>	

bers from other column :

3	Error : added in 3 again in
81	tens column.
19	This is sometimes due to bad
827	setting out of sums, some-
94	times to lapse of attention.
<u>1054</u>	

ying number twice :

28	Error : added in 2 twice in tens
103	column.
784	This is due either to lack of
9	consistency in the time at
<u>944</u>	which the carrying number is
	added or to retracing steps.

or numbers from column :

951	Error : omitted 8 in adding.
382	This is due to losing place in
467	column or to irregular habits
539	of adding. Practice needed
196	in checking answer down-
<u>2455</u>	wards.

SCHEDULE B

COMMON ERRORS IN SUBTRACTION

S. 1. *Omitted to allow for borrowing :*

Example	786	Error : omitted to allow for
	58	"borrowing," $8 - 5$ instead of
	<u>738</u>	$8 - 6$.

Not a few of these errors

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appear to be due to lapses, but in the early stages of subtraction many pupils go through a period of intermittent error in this direction. Oral working of examples decreases the error.

S. 2. Subtracted figures in top line (minuend) from those in bottom line (subtrahend) :

Example	316	Error : $6 - 7$ and $1 - 2$.
	27	Sometimes this is due to
	311	general ignorance of subtraction process, sometimes to

nature of figures in the two lines, sometimes to bad teaching. Thus oral examination of one group of backward pupils in a junior school showed that in such an example as the above they would say, "6 take away 7," that is, the top line mentioned first, with the result that in suggestible situations the wrong figures were subtracted.

S. 3. Subtraction of like numbers in minuend and subtrahend :

Example (a)	38	Error : $8 - 8 = 8$.
	8	Drill required on actual combinations of subtracting like numbers.
	38	

Example (b)	250	Error : $5 - 5 = 1$.
	49	
	211	

S. 4. Subtraction of "0" from a digit or a digit from "0" :

Example (a)	890	Error : $0 - 9 = 9$.
	889	This is a common error—that
	9	of writing in the answer the number to be subtracted from zero.

Example (b)	607	In general the difficulty seems to be one of adding 10 to 0.
	318	The child experiences difficulty in seeing that $10 - 0 = 10$, or in then using the 10 from which to subtract.
	319	

Example (c) 80
 57
 —
 30

S. 5. *Added instead of subtracting :*

Example 387
 196
 —
 421

S. 6. "Paying back" to the subtrahend when there was no "borrowing" :

Example 987 Error : 8 - 4 instead of 8 - 3.
 832 Give series of examples alter-
 — nating "borrowing" and no
 145 "borrowing."

SCHEDULE C

COMMON ERRORS IN MULTIPLICATION

M. 1. *Errors in tables :*

Example 7004
 8
 —
 48034

Errors : $4 \times 8 = 34$, $7 \times 8 = 48$.
 This error far outweighs any other in multiplication. It points to the difficulty and the need of making the basic multiplication facts absolutely automatic, through drill and games.

M. 2. *Errors in "carrying" numbers :*

Example (a) 874615
 9
 —
 7871495

Error : omitted to carry 4.
 Requires plenty of short sums involving a variety of "carrying."

Example (b) 56
 90
 —
 4940

Error : carried wrong number.
 In this case the number written down in the answer was carried.

Example (c) 95347
 6
 —
 562182

Error : $18 + 2 = 21$.
 Requires practice in combinations involving adding in multiplication.
 See Supplementary Test Y.

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M. 3. Errors in noughts in multiplier or multiplicand :

Example (a) 400 Error : $8 \times 0 = 8.$

$$\begin{array}{r} 8 \\ \hline 3288 \end{array}$$

Example (b) 90 Error : $90 \times 0 = 90.$

$$\begin{array}{r} 90 \\ 90 \\ \hline 8100 \\ 90 \\ \hline 8190 \end{array}$$

Example (c) 80 Error : position of figures,
100 800 for 8000 and $80 \times 0 = 80.$

$$\begin{array}{r} 800 \\ 80 \\ 80 \\ \hline 960 \end{array}$$

Example (d) 206 Error : omitted to carry
50 figure 3 after multiplying
10000 0 by 5.

M. 4. Errors in position of figures :

Example (a). Starting to multiply from the right :

$$\begin{array}{r} 34 \\ 22 \\ \hline 68 \\ 68 \\ \hline 136 \end{array}$$

Example (b). Starting to multiply from the left :

$$\begin{array}{r} 52 \\ 31 \\ \hline 156 \\ 52 \\ \hline 208 \end{array}$$

SCHEDULE D

• COMMON ERRORS IN DIVISION

D. 1. *Errors in basic combinations :*

Example (a) $\begin{array}{r} 411 \\ \hline 9 \end{array}$ Error : $27 \div 9 = 4$.
 $\begin{array}{r} 2799 \\ \hline 9 \end{array}$ Requires speed practice in division combinations, with and without remainders. Tests 4, 5 and Supplementary Test Z.

Example (b) $\begin{array}{r} 9 \text{ r. } 7 \\ \hline 9 \end{array}$ Error : $89 - 81 = 7$.

D. 2. *Omitted to carry figure :*

Example $\begin{array}{r} 32 \\ \hline 4 \end{array}$ Error : omitted to carry 1.

D. 3. *Remainder larger than divisor :*

Example $\begin{array}{r} 6 \text{ r. } 8 \\ \hline 7 \end{array}$ Error : 7×6 instead of 7×7 .
 Needs practice with basic combinations involving remainders.

D. 4. *Omitted " 0 " from quotient :*

Example (a) $\begin{array}{r} 701 \text{ r. } 3 \\ \hline 7 \end{array}$

Example (b) $\begin{array}{r} 375 \text{ r. } 3 \\ \hline 8 \end{array}$

D. 5. *Carried wrong number :*

Example $\begin{array}{r} 632 \text{ r. } 2 \\ \hline 5 \end{array}$ Error: $6 \times 5 = 30$, $34 - 30 = 4$, then carried 1 instead of 4.

D. 6. *Used same number in dividend twice :*

Example $\begin{array}{r} 3785 \text{ r. } 3 \\ \hline 8 \end{array}$ Error : 6 used in dividing into 56 and again to divide into 64; 4 used to divide into 64 and again to divide into 43.

CHAPTER IV

BACKWARDNESS IN ARITHMETIC

A KNOWLEDGE of the causes of backwardness in arithmetic is valuable to teachers not only from a remedial but also from a preventive standpoint. In arithmetic the motto "Forewarned is forearmed" is doubly significant in the early stages of teaching, for much can be done to prevent initial errors and ensure success if one is aware of possible pitfalls.

✓ Backwardness in arithmetic, like that in other elementary school subjects, is usually due to a plurality of causes. The clear-cut case of backwardness which can be ascribed to a single cause is comparatively rare ; the complex case which involves several causal factors is of common occurrence. We should, therefore, be prepared to find in most pupils backward in arithmetic several causal determinants of the paucity of their arithmetical attainments. For convenience of discussion in the following sections causes are considered separately under each of three headings :

- ✓ A. Environmental : within the home and within the school.
- ✓ B. Intellectual.
- ✓ C. Emotional.

A. ENVIRONMENTAL CAUSES OF BACKWARDNESS IN ARITHMETIC

From the outset it should be realised that arithmetic is a particularly difficult and abstract subject. Adults who have become absolutely automatic in their manipulation of addition, subtraction, multiplication and division facts sometimes forget that the young pupil is far from a stage of automatic accuracy. Most young children are puzzled by the arbitrariness and abstractness of the whole process of number, and it is only when they have had a sufficiently varied and comprehensive experience with the concrete that some understanding of number is developed. Initially the child builds up his elementary ideas of number through actual handling of objects. He first realises that one more thing than one makes two things ; but he remains on this level for a considerable time and although he may use other numbers he has no real understanding of their function. Gradually he extends the sphere of his understanding through common, everyday situations in which numbers are used, through games and through counting.

Counting is the basis of early number work and if the pupil is to understand unit values and group values he must have opportunity for much counting with different kinds of material. Although this early experience in number is important for most pupils if they are to bridge the gulf between the concrete and the abstract, it is essential to all dull and backward pupils if they are to make any progress in arithmetic. In so far as their mental age is below normal compared with others of the same chronological age, they require prolonged and intensified activity in concrete number situations before any attempt is made with formal work.

Two causes of backwardness in arithmetic are

thus apparent: namely lack of opportunity to acquire the requisite early number experience through handling and dealing with the concrete; and commencement of formal or abstract number work before the child has reached a mental level necessary for understanding relationships in an abstract medium.

(1) *Paucity of Pre-School Experience*

For children whose out of school life is limited and narrow, who have had little chance of counting, comparing, contrasting, measuring, weighing and sharing, and who have had little opportunity of seeing number applied to real life situations, it is necessary for the school to provide compensation for this limitation in their pre-school experiences. This can be, and is, done in modern infant schools by allowing such children to engage in numerous occupations that will give them the correct attitude towards number and the knowledge of fundamental ideas necessary to start formal work in the four rules. These occupations include movement with toys, building with blocks, use of form boards, counting with all sorts of materials in different shapes and forms, playing with scales involving the weighing of different materials, pouring water into vessels of varying sizes, doing simple woodwork, playing at shops, dairies, post offices and so on.

There are many such activities, but care should be taken to see that they all directly bear upon the formation of early number ideas. Infant schools in poor areas where formal work in number is postponed and where this kind of activity curriculum is substituted for the first six, nine or twelve months, according to the needs of the child, find that superior results are obtained when formal number is commenced at this later age.

(2) *Too early Commencement of Number with Dull Pupils*

The same treatment applies to dull pupils as to those from poor homes, except that there is a need to continue the activity work longer. It is obviously uneducational that children with mental ages of $3\frac{1}{2}$ to $4\frac{1}{2}$ (though of chronological ages 5 to 6 years) should be required to commence formal number as soon as they enter the infant school. It should be realised that "under present conditions many children at five are not mature enough to be taught anything except the names of the numbers. They should be placed in an environment where it is possible to work out their own salvation."¹ If this were done there would be less failure and confusion and fewer emotional barriers created in the early stages of arithmetic teaching.

(3) *Other Home Influences*

The child who, not over robust in health, gets insufficient sleep and perhaps insufficient food of a nourishing kind has neither the mental energy nor the power of concentration to maintain a consistent level of accuracy in an arithmetic lesson lasting forty, forty-five or even fifty minutes. Like the child who works early in the morning or late at night he soon fatigues and is easily distracted from his work, so that mastery of new steps and reproduction of old ones are often seriously affected.

For these pupils an enquiry into home conditions will often produce better results, particularly where hours of sleep are increased, than any actual individual assistance during arithmetic lessons. Furthermore, there is evidence that the arithmetic lesson for

¹ M. Drummond, *The Psychology and Teaching of Number*, p. 17. Harrap & Co. (3rd Impression), 1934.

young pupils and for dull pupils is not infrequently too long. The degree of inaccuracy, produced partly by boredom, partly by physical fatigue, is often so great after twenty-five minutes that it would be much better to have, with the younger and the duller arithmeticians in a school, two twenty or twenty-five minute periods a day than a single one of thirty-five or forty minutes.

(4) *Absence from School*

Absence from school, whether it be intermittent or prolonged, is one of the most important causes of disability in arithmetic. For progress in arithmetic is like ascending a staircase ; ascent to a new step is dependent upon the previous one, and if too many steps are missing then further progress is impossible. Furthermore, arithmetic is more susceptible to the influence of absences than any other school subject, for, whereas a child who has mastered the mechanics of reading can continue with his practice in out-of-school hours, this does not apply to arithmetic, in which new types and new steps are so numerous. Systematic and regular practice with graded examples, so necessary to automatise past steps and to consolidate new ones, is usually obtained only under class-room conditions. Some of the most pronounced cases of retardation in arithmetic in the junior school are pupils who, as a result of prolonged, or of numerous short spells of absence, have had insufficient practice in some operation or no instruction at all in certain essential steps.

Use of the Diagnostic Tests revealed pupils whose weakness in subtraction or in division was due to confusion in early steps, which resulted from ineffective treatment after absences. It is not only that the pupil actually misses certain work, but that consequent failure and loss of confidence play their

part in accentuating the effects of absence. Teachers should not fail to realise the feeling of dismay that absence from arithmetic lessons creates in some children. Adolescents to whom I have spoken can recall the loss of confidence that followed absence "when two new kinds of sums were taught." With many children, absent when new arithmetic material has been presented, there is need for definite individual work of a skilful kind to convince them that they can easily learn the new work and overcome their apparent handicap. The usual scanty explanations are insufficient.

(5) *Discontinuity*

(a) *Discontinuity between Infant and Junior Departments.*—Discontinuity between infant and junior departments is an accentuating factor in backwardness of some pupils. The somewhat backward child, who is still doing his number work largely through the use of concrete material, apparatus, games and the use of props, and who is unfortunate enough to be placed in a class in a junior school where all such aids are missing and where the work is almost completely formalised, mechanical and uninteresting, can soon develop into an apparently stupid arithmetician. It is a misguided practice that deprives pupils of 7 + and 8 +, particularly if they are not very bright, of apparatus and props as soon as they enter the junior school. Not a few of the dull ones require concrete material to help them with sums up to their tenth or eleventh year. Most children will use counters, dots and fingers only so long as they have need for them. The aim is to get the sums right, and if children cannot do this without the use of props then their use should be continued, for the success achieved in getting a sum right is the vital part of the whole lesson and the factor which will

most ensure the correct repetition of the same processes on a future occasion. No one prevents the adult from looking up forms of verbs or translations of words when he is learning a foreign language ; he hopes to render the necessary associations automatic through correct usage. Similarly with the child correct usage in initial stages is all-important. Prevent the use of aids that are essential to this and a confusion is engendered that inhibits normal expression of mental powers.

Miss Renwick, in discussing the need for the concrete, goes so far as to say that even in the senior school "a box of pieces of coloured cardboard could easily be prepared by the child and each pupil could conveniently keep his own supply of counters in readiness for his experiments with number."¹

(b) *Discontinuity between School and School or Area and Area.*—The pupil who moves from school to school or from area to area where methods of doing similar processes differ is likely to suffer from confusion if he experiences a second method in a fundamental operation before he has successfully mastered the first. No doubt variety and freedom in teaching are desirable, but in certain respects arithmetic is the subject least suited to diversity of method. For example, it has been proved beyond a shadow of doubt that the equal addition method of teaching subtraction results not only in greater accuracy, but in greater speed as well. If this is the case, then let us all agree to teach subtraction by this method ; the philosophic arguments of explanation should not deter us from the practical issues of the teaching problem. Similarly, with other methods and with the age levels at which certain processes are best taught, there is reliable evidence upon which

¹ E. M. Renwick, *The Case against Arithmetic—A Teacher's Evidence*, p. 117. Simpkin Marshall Ltd., 1935.

uniformity of procedure should be based. Unless this uniformity is in operation from area to area backwardness in arithmetic will increase, not decrease, and cases such as the following will become more common. Bessie was just beginning to learn compound multiplication and had been taught to multiply from the right. Before she had had sufficient practice to master the work she was transferred to a school where the pupils multiplied from the left. Although the new method was explained several times she continued to flounder for several weeks, making errors like these :

$$\begin{array}{r}
 35 \\
 18 \\
 \hline
 2800
 \end{array}
 \qquad
 \begin{array}{r}
 35 \\
 18 \\
 \hline
 280
 \end{array}
 \qquad
 \begin{array}{r}
 35 \\
 18 \\
 \hline
 350
 \end{array}
 \\
 \begin{array}{r}
 35 \\
 \hline
 35
 \end{array}
 \qquad
 \begin{array}{r}
 35 \\
 \hline
 35
 \end{array}
 \qquad
 \begin{array}{r}
 280 \\
 \hline
 280
 \end{array}
 \\
 \begin{array}{r}
 \underline{2835} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 \underline{315} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 \underline{3150} \\
 \hline
 \end{array}$$

The nature of the errors reveals the complete confusion that existed in the child's mind with regard to the position of the figures.

In conclusion, it would not be overstressing the point to advocate that there should be a greater measure of centralisation of suggestions concerning the major principles of methods in arithmetic. Even where this uniformity is observed with regard to methods used and ages at which particular processes are most effectively taught to pupils of different intellectual calibres, there is adequate opportunity for individual variation in selection of examples, application to everyday situations, practical work, use of concrete material, use of games and interesting drill methods.

(c) *Too rapid Promotion*.—It sometimes happens that a child, particularly bright on the verbal side

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but perhaps only average in arithmetical attainments, "skips" a class. This is right provided care is taken to link up the pupil's arithmetic knowledge from one class to the other and to ensure that he obtains adequate teaching and necessary practice with the arithmetic units that he would have done in the "skipped" class. Amongst my records are not a few cases of retardation or relative backwardness in arithmetic in comparatively bright pupils, where the original causal factor appeared to be a too rapid promotion in the subject. The possible development of this condition is excluded in a school where cross-classification or arithmetic sets are used.

(d) *Lack of Correspondence in Syllabuses from School to School.*—Where there is lack of correspondence in actual syllabus content from school to school pupils who transfer from one school to another are apt to miss vital units of work. The marked backwardness that some of these pupils exhibit after attendance at three or four different schools in the course of two years is further evidence that greater uniformity of procedure in sequence of, and age levels for, teaching the various arithmetical processes would decrease the amount of backwardness in arithmetic.

(6) *Teaching Methods*

Investigation with pupils backward in arithmetic shows that faulty teaching methods are contributory factors in the production of disability in arithmetic. One headmaster of a junior school is emphatic that "much backwardness in arithmetic is induced backwardness, that is, created during the child's school career." Of the attitudes and methods which are likely to contribute to or accentuate impoverished standards in arithmetic the following seem to be the most potent :

(a) *Over-explanation of Processes with Duller Pupils.*—There are some pupils, mainly the independent enquiring ones of supernormal intelligence, who must have, and who thrive on, explanations in arithmetic, but there are few pupils in "B" or "C" classes who require much explanation. What they require to know is how to get the sums right, and when they have learnt the method so thoroughly that the possibilities of getting the particular type of sums wrong are only the ordinary ones due to chance, then explanations might be attempted. Although most adults can divide one fraction by another or calculate square root, it is doubtful if more than 25 per cent of them can explain their methods of working. Why then should we burden children with unnecessary explanation in arithmetic?

A similar observation might be made regarding practical work. Sometimes the degree of correspondence between supposed and actual clarification of a process by means of practical exercises is not very marked. We find pupils cutting out papers and pasting them in position without any conception of which principle is being demonstrated—the activity simply becomes one of paper-cutting. This does not imply that practical work in arithmetic is of questionable value—on the contrary it is an absolute necessity for dull pupils and for those specifically backward in arithmetic—but it does imply that practical exercises should be such that they really help the child to understand the process and consequently to get his sums right.

(b) *Over-emphasis of Mechanical Work.*—Related to the last point is that of excessive mechanical work in arithmetic. Too much attention to purely mechanical exercises produces, particularly amongst dull children, an artificial division between arithmetic in school and arithmetic in everyday situations.

"Constant drill in the four rules, drill for its own sake, leads the children, especially if they are generally dull and backward in intelligence, to regard the four processes as four separate and distinct ideas entirely unrelated to one another. It is highly important that the problem or application aspect of number should be equally emphasised with children who are backward in arithmetic, since the dull mechanical work does in many cases increase their dislike of a subject that appears to them to be so useless."¹

(c) *A "too extensive" Syllabus.*—Arithmetic is a subject which has appealed to teachers, curriculum makers and text-book writers alike on account of its objectivity. Definite units of work can be selected, taught, applied and marked along clear-cut unequivocal lines. Furthermore, there still lingers the influence of a faulty faculty psychology which advocated that arithmetic teaches accuracy, trains in reasoning and encourages concentration. For these reasons arithmetic has tended to dominate the elementary school curriculum with the result that too much arithmetic or unsuitable arithmetic is sometimes taught. But of recent years improvement has taken place and pupils are no longer required to grapple with cube root, compound interest, practice by aliquot parts, compound proportion, and complex problems of the type in which they had to find how soon a bath of a certain capacity would be filled if it had five taps and three outlet pipes each pouring in or emptying, as the case might be, so much water per minute and all in operation at the same time. The curriculum for "A" and "B" pupils has been adjusted to some extent, but for less able pupils the work set is often beyond their abilities and their needs.

¹ E. Wheeler, *An Investigation into Backwardness in Arithmetic amongst London Elementary School Children*, p. 39. M.A. Thesis in the University of London Library.

Not a little backwardness in arithmetic amongst duller pupils would disappear if the curriculum were based on recognition of the fact that over 80 per cent of the problems with which adults have to deal in everyday life involve only the four fundamental processes with numbers under 100, fractions— $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{3}$ —and percentages. Nearly 90 per cent of the arithmetic is in connection with shopping.

But it is from the nature of the examples employed as well as from the syllabus that backwardness can be induced. Too often the pupil is asked to work exercises such as 96 tons 15 cwt. 3 qrs. 18 lbs. \times 74, $726826 \div 764$, £361 : 18 : $9\frac{1}{2} \times 96$ when more sums of a simpler kind, like those actually experienced in life, would form a better preparation for life.

(d) *Commencing a New Step before Previous One is mastered.*—Backwardness is in part due to the practice of allowing pupils to begin a new process or step before they have mastered preceding ones. For example, it is unsound to introduce a pupil to the difficulties of "carrying" in addition or "borrowing" in subtraction before he has attained a high degree of efficiency in the 100 basic addition and subtraction facts. The first difficulty of inaccuracy in the fundamental combinations only makes for confusion when the second and greater difficulties are experienced. There should be more consideration of individual levels reached in arithmetic and more consideration of the mental levels at which children are normally ready for particular processes.

To achieve the first aim the arithmetic lesson might be completely individualised; this is done where the class is small or where there is a very wide range of arithmetical attainments. But a measure of success is obtained where sections within the class or arithmetic sets are used.

The second point involves more research into

arithmetical abilities amongst elementary school pupils, but suggestive results on ages at which different processes should be commenced have been obtained from an American investigation. "The experiments show unmistakably that some of the topics in the school curriculum are commonly taught at points in children's growth which foredoom many to failure."¹

(e) *Bad Grading of Examples and an Endeavour to teach Two Similar but not Identical Types of Examples in the Same Lesson.*—These two aspects of arithmetic instruction are accentuating factors in backwardness. Careful grading of examples is essential to progress in arithmetic, yet one finds at times that after a new step has been taught insufficient simple examples are given to consolidate thoroughly the fresh knowledge. When a new step has been taken with a class it is advisable to give fifteen or twenty easy examples, each increasing very slightly in difficulty. As continued initial success is the dominant factor in enabling pupils to grasp the essentials of a new step, computational difficulties should be minimised in the early exercises, thereby allowing pupils to devote full attention to the method involved. Backwardness is produced when, with each new type of sum, the teacher gives only four or five simple examples and then plunges the pupils into relatively difficult variations—the brighter children succeed, the duller ones invariably flounder. There should always be a reasonable time interval between the teaching of new steps in a process.

Teaching similar but somewhat different types of examples in the same lesson is a procedure which confuses many rather dull children. For example, a class of pupils aged 10 to 11 years were taught in the

¹ C. Washburne, *Adjusting the School to the Child*, p. 39. World Book Co.

same lesson two easy types of proportionate division, namely :

- (A) Divide 10s. between 2 girls so that one has 3s. 6d. more than the other.
- (B) Divide £20 between 2 boys so that one has 4 times as much as the other.

Insufficient examples were given to clinch type A before type B was commenced, with the result that when miscellaneous examples were being worked the two types were confused. Thus examples such as :

“Divide 144 nuts between Tom and Harry so that Tom shall have 24 more than Harry”

would be solved by dividing 144 by 24.

B. INTELLECTUAL CAUSES OF BACKWARDNESS IN ARITHMETIC

(1) *Deficiency in General Intelligence*

The most obvious intellectual cause of disability in arithmetic is a defect in general intelligence. Arithmetic involves the use of symbols and, in so far as it is an abstract study making use of these symbols, it demands a considerable degree of general intelligence to succeed in it. Naturally the degree of general intelligence required is less when the examples are simple or when they are put in a mechanical form such as 40×8 , but where they involve a complex set of relationships then they demand a considerable degree of general intelligence. Thus a comparatively dull pupil may succeed with simple mechanical arithmetic but may fail hopelessly with relatively harder problem arithmetic. We have already indicated that general intelligence is the ability

- (a) to see relevant relationships between items of knowledge ;

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- (b) to educe correlates from these relationships, or, to put it in everyday language, to apply the relationships to new but similar situations.

Now arithmetic is shot through with relationships. For example, between the two numbers

6 and 2

numerous relationships may be perceived. Thus :

$$X_1 \quad 6 \times 2$$

$$X_2 \quad 6 \div 2$$

$$X_3 \quad 6 - 2$$

$$X_4 \quad 6 + 2$$

$$X_5 \quad 2 \text{ is ? of } 6$$

$$X_6 \quad 6 \text{ is ? times as great as } 2$$

These are all relationships perceived on the basis of the given fundaments, or items of knowledge. The second step, of educating a correlate, demands greater general intelligence, that is, to give the answers, in this case 12, 3, 4, 8, $\frac{1}{2}$, 3.

All kinds of relationships may be perceived between items of knowledge but only those relevant to the situation in hand are of any use at a particular time. The relationships cited and the correlates educed in the above example are, however, comparatively simple. In such an example as the following they are considerably more complex :

“ Three boys agree to divide some marbles so that the first shall have 20 more than the second and 30 more than the third. The number of marbles to be shared is 113. How many does each receive ? ”

It will be apparent from the foregoing very brief discussion that lack of general intelligence will account for some backwardness in arithmetic, particularly in problem-solving, in which the perception

of relationships demands not only memory but innate intellectual power.

(2) *Weak Memory for Numbers*

A defect in memory for numbers either immediate or delayed, sometimes both, is a common weakness of the backward arithmetician. It appears to be due to a combination of factors, chief of which are lack of interest in arithmetic, lack of confidence, and an innate deficiency in remembering numbers and number facts. The weakness shows itself in an inability to recall a sequence of numbers correctly—figures are omitted or transposed—so that often the pupil is most backward in oral or mental arithmetic. Sometimes this appears to arise from a definite weakness in visual imagery, that is, the pupil is unable to recall figures in his mind's eye. Thus in such an example as "39 horses cost £12 each, what

was the total cost?" the pupil cannot see $\frac{39}{\times 12}$ set out

on an imaginary blackboard.

It was noticeable that nearly all pupils weak in memory for numbers failed with the items like these (Nos. 35, 36 in Test 12, Diagnostic Tests) :

"Jack has to travel 100 yards along a straight road to school. He goes half way and then goes 25 yards along a side street and back to fetch a friend. How far does he walk to school?"

"A box is 6" long, 6" wide, 6" deep. How much string will I need to tie it up if the string goes round the box twice (once each way) and uses 3" for the knot?"

Experimental work shows that powers of visual imagery amongst children can be strengthened by

practice—encouraging them to close their eyes and picture things before them.

The pupil who has a weak memory for numbers is prone to rely upon props and temporary aids. He makes constant reference to tables, or can only recall a particular unit in a multiplication table if he goes through the entire table, or must actually put in his sums the carrying and borrowing figures. Hence his arithmetic is marked by a disproportionate number of errors involving the omission of figures in addition, in quotients, and in borrowing and carrying.

(3) *Weakness in Concentration*

Allied with a weakness in memory for numbers is a certain inefficiency in concentration or attention. Here again the cause may be specific, arising from early failure, dislike of the subject, too difficult or uninteresting material. There are some pupils who make foolish errors in their written arithmetic due to breaks in attention, but who do considerably better in oral arithmetic, particularly if the questions are attractively framed. On the other hand there are pupils for whom the repeated lapses in concentration have either a physical source or a psychological origin or in some cases a combination of both. The highly imaginative child or the pupil who is unwell or who suffers from the effects of semi-choreic instability, badly inflamed tonsils, decayed teeth, lack of sleep or food, will invariably show weakness in power of sustained attention. His mind will wander so that several figures in a column might be missed, or figures added in twice, or carrying figures left out or put in where unnecessary. Interest and success improve the way in which the child applies himself—arithmetic closely related to the pupil's interests and to everyday situations helps to increase intensity of attention during arithmetic lessons.

C. EMOTIONAL CAUSES OF BACKWARDNESS IN ARITHMETIC

Backwardness in arithmetic is due as much to emotional as to intellectual factors ; in fact, after working with backward pupils one is inclined to the conclusion that normal emotional reactions are *more important* than normal intellectual ones to progress in arithmetic. In many cases the confusion, the loss of self-esteem and self-confidence has been so great as almost to inhibit normal intellectual expression ; the children have become to all intents and purposes "number defectives." Detailed examination of such pupils reveals a number of related emotional troubles, sometimes as many as seven, rarely fewer than three, associated emotional reactions of an adverse type, such as loss of confidence, fear, anxiety, undue fantasy, cheating and compensatory misconduct. Early failure has produced confusion and later difficulties ; continued failure has bred a feeling of complete inadequacy in the whole subject. It is clear that these pupils require a measure of individual attention with quick success in arithmetic if they are to be helped.

Apart from the early experiences of some pupils—insufficient early number knowledge, absences, unfortunate teaching methods—that have contributed to their condition, there often are inherent characteristics in the make-up of the child which predispose him to failure in arithmetic. Thus amongst intelligent pupils, showing some degree of disability in arithmetic, one finds a section of over-emotional, impulsive, over-quick, careless children whose whole life is characterised by very little attention to detail, least of all the finer details of arithmetic, writing and spelling. These pupils have the intellectual power to do their work, but are so im-

pulsive that their output is very greatly reduced by inaccuracies. They have often finished their arithmetic first, but their speed is not paralleled by a corresponding level of accuracy. Their work displays all kinds of errors which show little uniformity from day to day. Reporting to the teacher after each sum is completed, and building up a habit of checking, minimises their errors to some extent. Such a procedure should be limited to individuals and never adopted with all the members of the class.

Somewhat the reverse in temperament to the pupils just described are those who exhibit a degree of instability, showing symptoms of nervousness, uncertainty and lack of persistence. A concrete case, taken from a number in my records, aptly illustrates the characteristics of this group. Kathleen D., aged $13\frac{4}{5}$, mental age $12\frac{5}{12}$, had an arithmetic age of only $9\frac{1}{2}$. Her age levels in the different processes and aspects were—

Addition	$9\frac{1}{2}$
Subtraction	$10\frac{3}{12}$
Multiplication	10
Division	$10\frac{1}{2}$
Mental	$8\frac{1}{2}$
Rules	$8\frac{1}{2}$
Problems	$7\frac{3}{4}$

The unevenness of her attainments is at once apparent, and these variations are also revealed in the sums she gets right and those she gets wrong. Thus Kathleen fails with—

$$\begin{array}{r}
 12 & 588 \\
 34 & \overline{)2344} \\
 10 & \\
 23 & \\
 \hline
 78 &
 \end{array}$$

but succeeds with—

$$\begin{array}{r} 7527\cdot 4 \\ 3698\cdot 6 \\ \hline 3828\cdot 8 \end{array} \qquad \begin{array}{r} 58603 \\ 15)879045 \\ \hline \end{array}$$

These variations in success and failure were indicative of Kathleen's fluctuating emotional attitudes. Her reactions towards her arithmetic disability varied between apprehension and apathy. Continued failure had produced a dislike of the subject, but the failure was in part due to her inability to apply herself consistently during arithmetic lessons. She was prone to day-dreaming and was particularly interested in drawing, dancing and reading. Her written English was on the whole good. Her meagre arithmetic attainments could be improved through systematic individual assistance, as later remedial work revealed.

Under the heading of emotional causes reference should be made to the atmosphere in which the arithmetic lesson is conducted. Progress in arithmetic is particularly susceptible to the influence of emotional states, and it is a fact well established, both from results of research and from class-room observation, that where a sympathetic, encouraging and stimulating atmosphere is preserved during arithmetic lessons much greater progress is made than where the atmosphere is one of censure, harshness and coercion. Some teachers fail to realise how easily a child may be upset by censure during an arithmetic lesson ; it is quite impossible for some children to add, subtract, multiply or divide accurately for some considerable time after they have been upset. Furthermore, many adults make errors in much of the same mechanical work set to pupils, yet the child is expected to reach a high standard of accuracy.

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Encouraging individual assistance together with a healthy atmosphere of competition and interest are vital nutritives to success in arithmetic.

In concluding this brief survey of causes of backwardness in arithmetic one might stress the importance of the early years in arithmetic teaching, the need for preventing confusion from arising in the mind of the child and the far-reaching psychological effects of failure.

SUMMARY OF CAUSES OF BACKWARDNESS IN ARITHMETIC

A. Environmental Causes of Backwardness :

- (1) Paucity of pre-school experience.
- (2) Too early commencement of number with dull pupils.
- (3) Other home influences.
- (4) Absence from school.
- (5) Discontinuity.
 - (a) Between infant and junior departments.
 - (b) Between school and school or area and area.
 - (c) Too rapid promotion.
 - (d) Lack of correspondence in syllabuses from school to school.
- (6) Teaching methods.
 - (a) Over-explanation of processes with duller pupils.
 - (b) Over-emphasis of mechanical work.
 - (c) A "too extensive" syllabus.
 - (d) Commencing a new step before previous one is mastered.
 - (e) Bad grading of examples and an endeavour to teach two similar but

not identical types of examples in the same lesson.

B. Intellectual Causes of Backwardness :

- (1) Deficiency in general intelligence.
- (2) Weak memory for numbers.
- (3) Weakness in concentration.

C. Emotional Causes of Backwardness :

- (1) Psychological effects of failure.
- (2) Temperamental disabilities.
 - (a) The impulsive child.
 - (b) The nervous child.
 - (c) The unsympathetic teacher

CHAPTER V

REMEDIAL WORK AND TEACHING METHODS

HAVING found, by means of the diagnostic tests, the exact levels of the backward pupils in the respective processes, we can commence remedial teaching at the correct stage. Should the diagnostic test results show, as may be the case with young pupils or with very dull ones, that a considerable number of the basic combinations are unknown and that the pupils have little idea of "carrying" or "borrowing," it is advisable to return to working very simple sums by use of concrete material. To begin with, such pupils should only attempt addition and subtraction in their new start. It is better to postpone multiplication and division, which involve a greater degree of abstraction, until some facility is reached in addition and subtraction. The pupils should be given large numbers of simple sums, carefully graded, and in this form :

$$\begin{array}{r} 2 & 4 & 1 & 0 & 1 & 2 \\ +2 & 1 & 5 & 0 & 4 & 3 \\ \hline - & - & - & - & - & - \end{array}$$

All the combinations should be covered many times, and answers derived from the use of counters where necessary. The object of this practice is to re-establish the child's confidence through continued success with easy examples and to give him the concrete experience which he evidently requires. The next step is to provide the pupil with plenty of practice without concrete aids, in order to render the fundamental facts as automatic as possible and

to increase the speed of his responses. For this, cards, games and testing sheets are useful.¹

DEVICES FOR RENDERING THE FUNDAMENTAL
NUMBER FACTS AUTOMATIC

The teacher, aided by the pupils, should make a large number of cards 3" by 2" and cut off the top left-hand corner of each so that the pupils will be able to keep the cards right way up. On one side of the card should be printed a number combination, and on the other side the combination with the answer, thus:

$$\begin{array}{r} 9 \\ + 6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ + 6 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 17 \\ - 8 \\ \hline \end{array} \qquad \begin{array}{r} 17 \\ - 8 \\ \hline 9 \end{array}$$

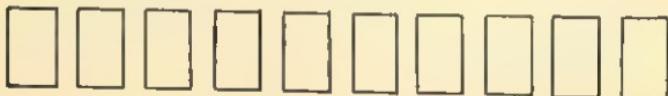
The cards should then be arranged in sets of increasing difficulty. The number facts in each process can be similarly prepared.

These cards can be used by pupils either individually or in groups for increasing accuracy and speed. For example, pupils can see how quickly they work through a set. The answer is recorded mentally and checked up by looking at the reverse side of the card, all unknown combinations being put on one side for additional drill. The value of this work lies in the interest it arouses in the pupils through the activity, and the fact that it both teaches and tests.

¹ Helpful remedial material for quick practice in the basic number combinations in the four rules is provided by *Set A, Schonell Practice Arithmetic Cards*. Oliver & Boyd.

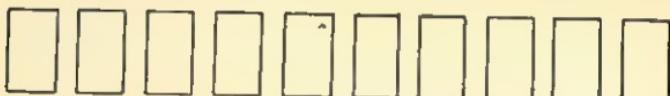
Once a certain degree of mastery has been attained pupils should be encouraged to test one another, A's test B's and B's test A's. Several sets can be worked through each morning in a very short time and with a very considerable improvement in accuracy in the fundamental facts in the four rules. Pairs of pupils can also play games of "snap" with one set. A useful game for a group of three backward children is "Pairing." Two pupils (the players) have each a full complement of cards of the same set, while a third pupil (the dealer) has a set of cards, previously prepared, bearing only the answers to combinations used in the set. The players spread out their cards in rows before them thus :

Player A



Dealer

Player B



The dealer then places a card in the centre (for example, one bearing the number 15), and the first player to cover it with the card $9 + 6$ scores a point. The first pupil to score 20 points wins the game.

After pupils have made up their own multiplication and division tables from counters, cards can be prepared for practice. Suppose the 8 times table

requires practice, then A and B cards, 6" by 4", are prepared as follows :

A CARD

9	2	8	3	9	7	6	4	0
3	2	12	0	9	8	9	10	1
11	9	8	7	6	12	9	8	7
6	5	12	4	11	9	8	10	4
6	3	2	1	8	9	6	7	12

B CARD

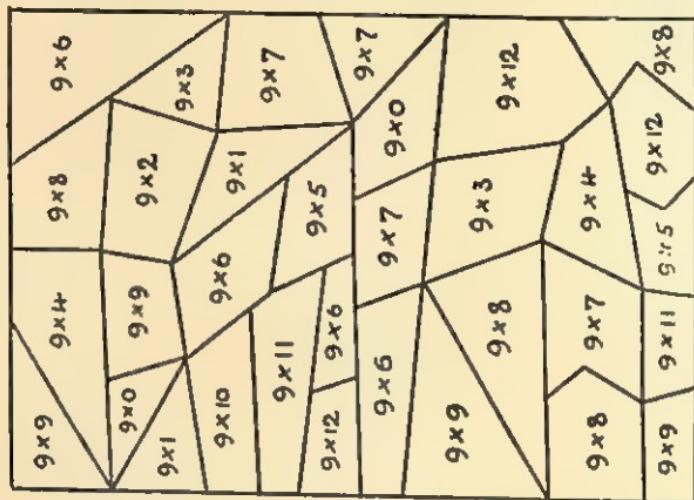
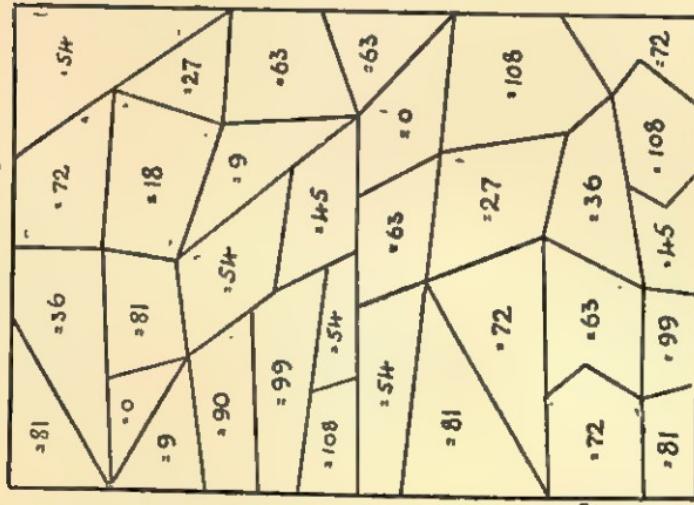
8	8	8	8	8	8	8	8	8	8	8	8
1	2	3	4	5	6	7	8	9	10	11	12
8	16	24	32	40	48	56	64	72	80	88	96
72	16	64	24	72	56	48	32	0			
24	16	96	0	72	64	72	80	8			
88	72	64	56	48	96	72	64	56			
48	40	96	32	88	72	64	80	32			
48	24	16	8	64	72	48	56	96			

The children then work in pairs, those on the left of the seat being designated A, those on the right of the seat B. Pupil A takes card A and goes as quickly as possible through the process of multiplying by 8 the units on the card. Pupil B has card B which shows the answers and a check table at the top. Cards are then exchanged.

A modification of multiplication cards greatly enjoyed by normal as well as backward pupils are jig-saw cards. These are cards about 12" by 8" made of semi-stiff cardboard, and they are prepared as follows. Both cards are ruled with clear black lines into a number of irregularly shaped figures, one card being a replica of the other, but care being taken to see that no two figures on the same card

are alike in shape. One card is first ruled and then pricked through on to the other card to ensure that

CARD A
(To be cut into pieces)



the shapes on both are identical. On card B the elements of the table are written in varied order, one on each of the figures, while the correct product is

written on the corresponding figure on the other card. The form of the cards is reproduced on p. 90.

Card A, complete with the products, is then cut up into shaped pieces. The pupil takes these pieces, on which the products are written, and places them in their correct positions on card B. It is not possible for him to make an error because each piece has only one position on card B, so that testing as well as teaching takes place. When card B is full the pupil simply lifts off the small pieces, repeating the multiplication fact, as he does so, to his partner or teacher. Speed tests and records of achievement from day to day can be kept for all such material.

Another useful device for backward pupils is the number chart for variation in the four processes. The chart is compiled as follows :

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

All kinds of number facts with useful higher-decade additions can be taught and tested from such a chart.

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by taking rows of figures in different ways and different positions.¹ For example, each of the numbers 1 to 9 can be in turn added to all other numbers, thus :

$$\begin{aligned} & 11 + 5 ; \quad 21 + 5 ; \quad 31 + 5 ; \quad 41 + 5 ; \quad \text{etc.} \\ & \text{or } 13 + 9 ; \quad 23 + 9 ; \quad 33 + 9 ; \quad 43 + 9 ; \quad \text{etc.} \end{aligned}$$

thus giving much practice on higher decade addition.

Pupils not only enjoy learning their number facts by these devices through the interest and activity evoked, but they learn them much more effectively.

Supplementary to the card practice, and as a midway step between concrete aids and abstract number work, the pupils should be given plenty of simple mental arithmetic in which the number facts are applied to everyday situations. All kinds of interesting material involving the various number combinations can be used for this work. The more stimulating the talk about the material or the situations the more vivid is the imagery aroused and the more interested are the pupils, so that they do their arithmetic without realising it is an arithmetic lesson.² For example, a variety of number facts can be worked through in connection with numerous topics such as—

- (a) Birds sitting on telephone wires—changing positions to give different combinations.
- (b) Clusters of cherries on a tree—picking them, giving some away, etc.
- (c) Windows in a house, panes of glass in each window, etc.
- (d) Milk bottles in a tray, delivery, rearranging, etc.

¹ For further details see M. Punnett, *The Groundwork of Number*, p. 81 onwards.

² For many useful suggestions concerning this aspect of arithmetic teaching to young pupils and backward arithmeticians I am indebted to Mr J. A. Barrow, H.M.I.

- (e) Rose bushes in a garden, roses on the bushes, picking them, arranging them in vases, etc.

When the backward pupils have reached a fair degree of accuracy in the fundamental processes—this can be determined from a re-test with the Diagnostic Tests 1 to 4—then a start can be made once again with graded examples, taking care to proceed by carefully considered steps and to give adequate practice in the simple problem aspects as well as in the mechanical ones.¹

IMPROVEMENT IN PROBLEMS

The task of eliminating backwardness in mechanical arithmetic is difficult enough, but it is doubly so when we deal with problem-solving, progress in which is for most children a matter of constant practice with stereotyped examples each characterised by its particular clues and catchwords. The difficulty arises from two sources ; firstly, ability to solve problems is very closely related to level of general intelligence, hence dull pupils will almost invariably experience difficulty with problem arithmetic ; and secondly, this ability is dependent upon numerous other factors such as reading, memory and computational accuracy.

The essentials in problem-solving might be enumerated as follows :

J. Intelligent reading of the problem.

¹ A large number of examples, carefully graded in easy steps and in sets of increasing difficulty, suitable not only for normal but for backward pupils, are to be found in *Right from the Start Arithmetics*, Books I and II (3rd edn.), by F. J. Schonell and S. H. Cracknell (Oliver & Boyd, 1940). These books also contain counting exercises to give practice in the higher decade addition facts, practical work and plenty of simple problems, all suited to pupils of varying intellectual powers.

2. Technique of attack ; analysis and arrangement of data.
3. Seeing relationships between the data.
4. Seeing an analogy with similar problems.
5. Selecting and reproducing the process.
6. Accurate computation.
7. Approximate checking of the result.

The discerning teacher will see that, in certain of these steps, namely 3, 4, 5, it will be difficult to bring about improvement, but in others improvement will take place if definite attention is given to the specific step. Thus, as problems contain both verbal and numerical materials which are read in somewhat different ways, noticeable improvement in problem-solving follows practice in problem-reading. Pupils should be trained to make a first reading to understand the principle involved, a second to appreciate the details of the numbers, and a third, attending to both principle and numbers. This procedure minimises premature generalisation, incorrect transcription of numbers and loss of confidence experienced when large numbers are met with at the beginning of a problem.

Improvement in phases 3, 4 and 5 must be brought about by replacing reasoning power and novelty, as far as possible, by memory and imitation, i.e. by a careful division of problems into types, then a treatment of a variety of sub-types.

At times the pupil is handicapped in problem-solving because insufficient practice is devoted to easy one-step problems of an everyday nature that cover all combinations of addition, subtraction, multiplication and division. Junior pupils benefit from considerable practice in these simple problems, examples of which are given on the next page :

Example 1. (Addition and subtraction.)

A boy is in school for 3 hours in the morning.

Assembly takes 15 minutes, arithmetic 40 minutes, milk 5 minutes, playtime 15 minutes and the rest of the morning is taken up by a drawing lesson. How long is the drawing lesson?

Example 2. (Multiplication and addition.)

On Monday Mother took $2\frac{1}{2}$ pints of milk and on each of the other days, except Sunday, she took $1\frac{1}{2}$ pints. On Sunday she took 3 pints. How many pints did she take for the whole week?

Example 3. (Multiplication and subtraction.)

Last month the gas cost Mother 2s. 7d. a week, but this month it cost her only 1s. 11d. a week. How much more did it cost her last month? (4 weeks = one month.)

Example 4. (Multiplication and division.)

There are 5 apples to a pound. Mother has 6 pounds to last 3 days. How many apples should she use each day?

Example 5. (Subtraction and addition.)

Fresh butter is 1s. 4d. a pound and salt butter is 3d. a pound less. How much must I pay for a pound of salt and a pound of fresh butter?

Example 6. (Addition and division.)

Mother had 3s. 6d. in silver, 2d. in pennies, as well as 4d. in halfpennies. This was just enough to buy a joint of 4 pounds of beef. How much a pound was the beef?¹

¹ See Book I of *Right from the Start Arithmetics* (Oliver & Boyd) for 160 of such problems which cover all possible combinations. The problems, which are suitable for normal pupils of 8 and 9 years of age, and for backward ones of all ages, are graded, while those combinations most commonly used in everyday life receive additional emphasis.

The teacher should remember that an inductive-deductive approach is the best for general problem-solving, but for particular types of problems taken one at a time the deductive seems superior.

GENERAL CONSIDERATIONS

General methods which have a bearing on backwardness in arithmetic are as follows :

- (1) Where pupils in ordinary register classes show a considerable spread in arithmetical attainments the backward scholars receive most effective instruction by classification throughout the school, where it is possible, into subject sets.¹
- (2) There should be constant application of arithmetical material to actual life situations. Arithmetic for backward boys and girls in a senior school should be, for the final year of their course, almost entirely based on social activities worked out in a semi-project form. Shopping ; cost of clothing ; exercises in judgment of planning and cutting out materials ; costs of foods ; diets ; balance of diet ; furnishing rooms with costs of furniture, linoleum, curtains, etc., given ; measurement and costs of wood for simple home articles ; daily, weekly and quarterly accounts—these and allied topics form suitable functional centres round which to organise a practical arithmetic curriculum for boys and girls in senior schools.
- (3) Work on money, length, area, weight and

¹ For additional details see *The Education of Backward Children*. Reprint from *The Year Book of Education*, 1936, chapters 4 to 7, by F. J. Schonell. Evans Bros., London.

- capacity should always be preceded by a considerable amount of practical work in which pupils handle the actual units that they are going to use in formal exercises. Different activities can be organised to give the pupil a thorough understanding of 1d., 3d., 6d., 1s., of 1 oz., $\frac{1}{4}$ lb., $\frac{1}{2}$ lb., $\frac{3}{4}$ lb., of 1 inch, 1 foot, 1 yard, keeping to a minimum, in the initial stages, the reference to actual number. It is the concepts of the units and their equivalent values that should be understood through this preliminary work.¹
- (4) Every attempt should be made to break down the traditional division between the various topics in arithmetic. This can be begun by assisting pupils to see the relationship between the four rules, while such linking as half-pennies and farthings with first lessons in fractions, and of fractions, in general, with decimals and percentages, should characterise the whole syllabus.
 - (5) With pupils backward in arithmetic it should be a major aim to motivate all work as strongly as possible. Normal pupils often receive sufficient incentive from the mere manipulation of numbers and the success they achieve from correct solution of examples, but for those who are dull or who have failed in arithmetic there is an emotional barrier which must be surmounted by the stimulus of the situation demanding the use of arithmetic. In addition to the usual ways of motivating work for pupils it will be found that individual progress charts, speed tests,

¹ For a suggestive discussion of this approach see E. M. Renwick, *The Case against Arithmetic*, pp. 148-155. Simpkin Marshall Ltd.

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plenty of oral arithmetic related to interesting topics (the material for which has been collected by the children in the course of daily activities, e.g. the number of trams, buses, cars passing a certain point in a certain time or the number of people in a picture theatre queue, etc. etc.), all aid in making arithmetic an interesting lesson.

- (6) All useless processes and large numbers should be eliminated.

In conclusion it should be stressed that simple sums are of vital importance in the early stages of remedial work with backward children. Innately dull pupils should only be expected to make attainments commensurate with their mental powers. A dull boy of 11, with a mental age of 9, will always be backward compared with his normal 11-year-old companions, but provided he attains a 9-year-old level in arithmetic this is all that can be expected of him.

So far as the diagnosis indicates, differentiation should be made between pupils who have failed entirely to assimilate the basic combinations, those who have acquired them but who show marked, intermittent inaccuracy in them, and those who, while showing skill in the fundamental facts, are unable to apply their knowledge in certain types of exercises. Pupils weak in learning the fundamental facts should be differentiated—and given the additional aids they require—from those who, through absence, have not had a fair chance of learning the facts. Repetition of the same types of error seems to indicate that the pupil has not mastered the facts or the process ; inconsistency of error rather indicates lack of continued attention.

Finally, it cannot be too strongly emphasised

that, if pupils backward in arithmetic are to be helped effectively, they must receive some measure of individual attention in their difficulties. For it is the emotional attitude of these children towards their backwardness which is the deciding factor in determining whether many of them will improve or not. A sympathetic, co-operative attitude on the part of the teacher, together with a certain amount of initial success on the part of the pupil, will do more to overcome disability than any other measures.

NOTE

A useful modification of the jig-saw arithmetic cards (described on pages 89-91) has recently been suggested to me by Mr R. K. Robertson, M.A. He found that in his work with backward arithmeticians who had made a little progress the base card (Card B) should not have lines on it, but merely the numbers. Pupils are thus forced to put each irregularly shaped card over the correct number without guidance from lines. At the same time the fact that all pieces of cardboard should fit together like a jig-saw puzzle provides an automatic check.

APPENDIX 1

TABLE B

AVERAGE SCORES IN GIVEN TIMES FOR TESTS 6 TO 12 FOR AGE GROUPS 7 TO 14 YEARS

Tests	Time allowed	Ages in Years							
		7	8	9	10	11	12	13	14
Test 6	6 mins.	15	23	32	41	46 (43)	50 (45)	52 (47)	54 (49)
Test 7	5½ mins.	8	15	22	31	38 (31)	42 (35)	46 (39)	52 (43)
Test 8A	7 mins.	7	12	19	24	29 (24)	34 (27)	38 (31)	40 (34)
Test 8B	7 mins.	2	5	7 (4)	9 (6)	10 (7)	10 (8)
Test 9	5 mins.	5	11	17	24	30 (24)	35 (27)	38 (30)	41 (32)
Test 10	9 mins.	10	18	23 (18)	27 (20)	30 (23)	33 (25)
Test 11	15 mins.	2	7	11 (8)	14 (9)	17 (11)	19 (13)
Test 12	10 mins.	3	9	14	19	24 (24)	28 (27)	32 (29)	34 (31)

The figures in brackets, for ages 11 to 14, refer only to scores of pupils from Senior Schools or non-selective Central Schools. These are lower, as some pupils of these ages have already gone to Secondary and/or selective Central or Higher Schools.

SUPPLEMENTARY TESTS¹

SUPPLEMENTARY TEST X

100 Important Higher Decade Addition Combinations

	(a)	(b)	(c)	(d)	(e)
A.	37 + 2 =	11 + 8 =	34 + 3 =	31 + 8 =	30 + 7 =
B.	30 + 6 =	31 + 5 =	23 + 6 =	22 + 3 =	22 + 4 =
C.	11 + 7 =	23 + 5 =	33 + 1 =	33 + 4 =	11 + 4 =
D.	33 + 6 =	12 + 6 =	11 + 3 =	22 + 6 =	30 + 9 =
E.	13 + 4 =	11 + 1 =	10 + 9 =	34 + 5 =	11 + 5 =
F.	21 + 8 =	11 + 6 =	12 + 7 =	22 + 5 =	26 + 3 =
G.	23 + 4 =	31 + 4 =	23 + 3 =	22 + 7 =	20 + 9 =
H.	31 + 2 =	23 + 2 =	10 + 6 =	14 + 8 =	17 + 5 =
I.	23 + 7 =	17 + 3 =	22 + 8 =	19 + 6 =	23 + 9 =
J.	15 + 5 =	13 + 7 =	15 + 6 =	29 + 5 =	16 + 8 =
K.	14 + 7 =	24 + 9 =	25 + 5 =	28 + 8 =	23 + 8 =
L.	17 + 4 =	21 + 9 =	29 + 3 =	29 + 9 =	25 + 7 =
M.	26 + 5 =	19 + 2 =	19 + 8 =	29 + 4 =	29 + 6 =
N.	22 + 9 =	19 + 4 =	14 + 9 =	36 + 4 =	18 + 9 =
O.	15 + 9 =	17 + 7 =	17 + 9 =	25 + 6 =	26 + 8 =
P.	16 + 7 =	24 + 8 =	25 + 9 =	26 + 7 =	17 + 8 =
Q.	29 + 7 =	29 + 8 =	36 + 9 =	19 + 3 =	28 + 9 =
R.	11 + 9 =	25 + 8 =	28 + 7 =	26 + 6 =	12 + 9 =
S.	13 + 8 =	27 + 9 =	12 + 8 =	29 + 2 =	19 + 7 =
T.	16 + 9 =	19 + 9 =	13 + 9 =	17 + 6 =	15 + 8 =

¹ These tests, together with the basic number combinations in Tests 1 to 5 of the Diagnostic Tests, may be obtained in card form for individual pupil practice. See Schonell *Practice Arithmetic Cards, Sets A and B*. Oliver & Boyd.

SUPPLEMENTARY TEST V

160 Important Addition Combinations required for Carrying
in Multiplication

	(a)	(b)	(c)	(d)	(e)
1.	40 and 2 =	20 and 4 =	12 and 5 =	63 and 5 =	24 and 4 =
2.	81 and 2 =	16 and 2 =	32 and 7 =	64 and 2 =	40 and 6 =
3.	42 and 2 =	14 and 4 =	56 and 3 =	72 and 4 =	21 and 5 =
4.	81 and 6 =	72 and 7 =	54 and 3 =	12 and 3 =	32 and 6 =
5.	21 and 1 =	10 and 1 =	12 and 4 =	40 and 7 =	72 and 5 =
6.	21 and 6 =	81 and 5 =	63 and 3 =	14 and 3 =	36 and 3 =
7.	40 and 5 =	21 and 4 =	42 and 5 =	72 and 6 =	27 and 2 =
8.	63 and 6 =	24 and 5 =	14 and 5 =	30 and 3 =	32 and 3 =
9.	81 and 3 =	64 and 5 =	35 and 4 =	63 and 4 =	54 and 5 =
10.	81 and 8 =	24 and 3 =	36 and 2 =	35 and 2 =	72 and 3 =
11.	25 and 4 =	30 and 5 =	24 and 2 =	54 and 4 =	64 and 3 =
12.	81 and 7 =	54 and 2 =	15 and 4 =	42 and 6 =	72 and 2 =
13.	27 and 1 =	32 and 4 =	32 and 5 =	18 and 3 =	45 and 3 =
14.	42 and 3 =	20 and 2 =	63 and 2 =	45 and 4 =	18 and 5 =
15.	49 and 2 =	16 and 4 =	54 and 6 =	14 and 6 =	18 and 4 =
16.	48 and 6 =	64 and 6 =	49 and 1 =	36 and 4 =	28 and 3 =
17.	18 and 3 =	28 and 2 =	18 and 2 =	27 and 3 =	27 and 4 =
18.	56 and 4 =	16 and 6 =	48 and 2 =	24 and 6 =	49 and 5 =
19.	27 and 6 =	18 and 8 =	16 and 5 =	16 and 7 =	36 and 6 =
20.	35 and 6 =	18 and 6 =	27 and 7 =	18 and 7 =	48 and 3 =
21.	54 and 8 =	28 and 4 =	45 and 8 =	36 and 5 =	24 and 7 =
22.	72 and 8 =	45 and 7 =	49 and 4 =	49 and 6 =	45 and 5 =
23.	36 and 7 =	28 and 5 =	35 and 5 =	56 and 7 =	27 and 5 =
24.	63 and 8 =	64 and 7 =	45 and 6 =	49 and 3 =	27 and 8 =
25.	56 and 6 =	28 and 6 =	56 and 5 =	64 and 4 =	48 and 4 =
26.	48 and 5 =	36 and 8 =	48 and 7 =	63 and 7 =	54 and 7 =
27.	33 and 8 =	22 and 8 =	84 and 6 =	96 and 8 =	84 and 8 =
28.	33 and 9 =	84 and 9 =	96 and 6 =	84 and 7 =	96 and 9 =
29.	96 and 7 =	48 and 8 =	48 and 9 =	36 and 9 =	24 and 9 =
30.	24 and 8 =	77 and 8 =	66 and 9 =	99 and 9 =	108 and 6 =
31.	88 and 8 =	108 and 8 =	88 and 9 =	44 and 6 =	55 and 7 =
32.	108 and 7 =	108 and 9 =	99 and 5 =	77 and 9 =	72 and 9 =

SUPPLEMENTARY TEST Z

105. Difficult Division Combinations with Remainders

	(a)	(b)	(c)	(d)	(e)
A.	4)25	3)34	6)55	5)56	7)50
B.	4)37	6)37	6)67	8)73	7)78
C.	7)64	9)82	6)73	8)49	9)73
D.	9)20	8)20	9)40	6)50	8)50
E.	6)20	3)20	9)50	8)30	7)20
F.	4)30	6)40	9)60	8)70	9)30
G.	7)30	7)40	5)19	7)60	9)80
H.	9)43	9)61	7)34	7)62	9)26
I.	9)32	6)53	8)31	9)31	9)33
J.	9)23	9)42	7)32	8)55	9)21
K.	7)52	7)61	7)31	6)51	7)33
L.	8)51	9)51	9)71	8)23	9)24
M.	9)41	8)21	8)38	9)53	8)22
N.	8)63	8)52	5)39	9)88	9)34
O.	9)22	5)49	7)48	6)57	4)38
P.	7)54	8)67	9)35	6)47	7)53
Q.	8)54	7)51	6)23	8)53	9)44
R.	6)46	6)17	9)87	4)47	7)55
S.	8)61	6)41	9)25	4)43	8)68
T.	7)19	9)47	7)41	5)62	6)22
U.	6)21	6)52	9)52	9)62	8)71

APPENDIX 2

ANSWERS TO THE TESTS

(i) ANSWERS TO THE SCHONELL DIAGNOSTIC TESTS 1-12

Answers to the items in the twelve tests are set out below. When pupils are correcting their own results teachers should remember to call answers across the pages, that is in the order in which the sums have been worked.

TEST 1. ADDITION

100 Basic Addition Facts

	(a)	(b)	(c)	(d)	(e)
A.	2	0	4	3	4
B.	2	4	6	10	5
C.	7	4	8	8	7
D.	8	9	6	5	14
E.	7	5	5	2	12
F.	7	16	11	10	9
G.	11	10	18	3	7
H.	1	6	8	6	4
I.	10	10	10	9	8
J.	11	6	9	3	3
K.	5	6	9	9	9
L.	1	8	9	8	7
M.	5	7	7	8	8
N.	9	12	6	10	12
O.	9	10	11	11	10
P.	11	11	12	13	12
Q.	13	11	17	12	13
R.	12	14	13	14	15
S.	14	15	15	17	16
T.	14	16	13	13	16

TEST 2. SUBTRACTION

100 Basic Subtraction Facts

	(a)	(b)	(c)	(d)	(e)
A.	1	0	2	1	0
B.	2	0	4	0	7
C.	0	1	2	1	1
D.	3	5	8	0	1
E.	0	6	3	0	3
F.	4	4	0	1	2
G.	1	9	2	6	2
H.	4	3	6	6	5
I.	3	9	3	5	8
J.	6	4	4	0	1
K.	6	2	3	2	8
L.	7	5	5	7	3
M.	9	4	4	7	3
N.	5	2	6	1	3
O.	7	8	4	8	2
P.	5	7	7	7	9
Q.	8	7	8	5	9
R.	6	6	5	9	8
S.	8	9	9	7	6
T.	8	9	5	4	9

TEST 3. MULTIPLICATION

100 Basic Multiplication Facts

	(a)	(b)	(c)	(d)	(e)
A.	3	4	7	2	4
B.	5	10	6	16	5
C.	4	6	8	6	18
D.	20	14	16	6	10
E.	3	9	9	15	12
F.	2	12	12	25	24
G.	18	36	8	20	18
H.	30	8	12	15	27
I.	32	30	40	35	24
J.	45	1	48	54	42
K.	36	21	28	24	7
L.	14	18	16	9	0
M.	8	24	21	0	35
N.	81	0	45	0	0
O.	0	0	32	0	28
P.	64	0	72	72	0
Q.	48	0	42	0	0
R.	56	63	0	54	49
S.	0	0	56	40	0
T.	63	27	0	0	36

TEST 4. DIVISION
90 Basic Division Facts

	(a)	(b)	(c)	(d)	(e)
A.	2	5	3	3	2
B.	5	4	7	4	5
C.	6	8	2	4	3
D.	2	6	8	4	3
E.	6	7	7	4	5
F.	7	6	3	7	6
G.	5	5	8	2	9
H.	9	9	4	8	2
I.	3	6	8	8	9
J.	1	1	9	1	1
K.	9	1	7	1	
L.	2	3	3	2	4
M.	2	8	9	5	5
N.	0	7	4	0	3
O.	5	0	8	0	9
P.	0	6	0	4	0
Q.	6	9	0	8	7
R.	0	6	1	7	1
S.	1				

TEST 5. MISCELLANEOUS

100 of the most difficult Addition, Subtraction, Multiplication and Division Facts

	(a)	(b)	(c)	(d)	(e)
A.	11	7	42	12	6
B.	9	0	5	7	8
C.	12	4	63	13	17
D.	13	9	0	24	7
E.	0	6	8	54	56
F.	6	11	11	1	
G.	13	0	7	9	0
H.	13	0	9	54	49
I.	7	0	27	40	5
J.	6	12	14	14	15
K.	8	5	0	8	1
L.	9	6	8	42	56
M.	0	15	9	9	0
N.	9	36	0	9	7
O.	77	7	8	72	
P.	48	121	8	132	120
Q.	5	84	9	96	11
R.	6	110	88	11	10
S.	99	7	12	144	132
T.	4	60	9	12	10
					108

TEST 6. GRADED ADDITION

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
A.	17	19	18	19	25	29	26	21
B.	97	87	96	58	168	558	787	669
C.	28	21	23	26	64	64	95	76
D.	118	159	178	131	40	87	93	96
E.	1008	248	957	1579	125	124	116	151
F.	173	210	179	173	1394	1771	1253	1394
G.	222	312	609	1950	924	1565	1024	2535
H.	51							
I.	62							

TEST 7. GRADED SUBTRACTION

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
A.	95	53	83	30	23	56	68	75
B.	131	155	256	235	4	1	6	2
C.	69	58	39	78	15	5	39	19
D.	313	518	254	728	289	505	9	88
E.	50	100	23	14	109	201	129	1
F.	62	156	374	191	69	555	2678	1314
G.	105	309	508	707	684	698	596	97

TEST 8A. GRADED MULTIPLICATION

	(a)	(b)	(c)	(d)	(e)	(f)
A.	88	155	126	455	1,269	2,448
B.	3,555	1,686	360	909	3,200	45,050
C.	162	85	112	133	684	688
D.	576	348	936	742	88,550	56,032
E.	4,176	46,156	1,144,164	7,871,535	748	1,612
F.	7,332	3,648	7,760	5,040	2,370	5,040
G.	8,100	5,076,000	8,000	70,000	28,497	40,500

TEST 8B. GRADED MULTIPLICATION

H.	55,680	17,545	15,096
I.	76,128	10,300	36,420
J.	316,404	218,154	145,340
K.	2,302,951	4,283,400	

108 INDIVIDUAL DIFFICULTIES IN ARITHMETIC

TEST 9. GRADED SIMPLE DIVISION

	(a)	(b)	(c)	(d)
A.	11	42	32	111
B.	341	211	232	1,213
C.	302	403	210	230
D.	200	300	100	200
E.	303	906	904	607
F.	6 r.3	6 r.5	8 r.8	8 r.5
G.	7 r.1	24 r.1	9 r.8	9 r.3
H.	31 r.1	41 r.3	21 r.2	21 r.2
I.	41 r.3	63 r.3	87 r.1	51 r.8
J.	82 r.2	124 r.1	692 r.2	796 r.1
K.	3,705 r.3	5,071 r.2	7,001 r.3	34,207 r.5

TEST 10. LONG DIVISION (EASY STEPS)

	(a)	(b)	(c)	(d)
A.	2	3	2	4
B.	2 r.2	3 r.3	2 r.1	4 r.6
C.	2 r.3	4 r.17	2 r.12	3 r.5
D.	2 r.8	2 r.9	3 r.3	2 r.8
E.	6	4	4	3
F.	3 r.10	2 r.1	4 r.4	3 r.2
G.	21	23	28	34
H.	78 r.1	64 r.3	81 r.3	63 r.7
I.	70	50	50	50

TEST 11. LONG DIVISION (HARDER STEPS)

	(a)	(b)	(c)	(d)
A.	20 r.3	32 r.9	62 r.3	70 r.6
B.	3	4	6	4
C.	9	7 r.10	7 r.23	9 r.3
D.	47	34	65 r.22	41
E.	459	3/9 r.13	189	608
F.	708	39 r.9	840 r.19	807 r.8

TEST 12. GRADED MENTAL ARITHMETIC

1. 12	9. 1/8	17. 2/-	25. 15	33. £1 : 2 : 6
2. 12	10. 16	18. 1/7½	26. 3½d.	34. 48 inches
3. 10	11. 21	19. 2 lbs.	27. 42	35. 150 yards
4. 14	12. 35	20. 3/6	28. 1½d.	36. 51½ inches
5. 12	13. 9	21. 3/6	29. 3½d.	37. 1/10½
6. 6	14. 12	22. 6/1½	30. 15	38. 34
7. 18	15. 47	23. £1 : 13 : 0	31. 80	39. 6.40 P.M.
8. 2½d.	16. 72	24. 3/1	32. 10.55 A.M.	40. 1/2

(ii) ANSWERS TO SUPPLEMENTARY TESTS

SUPPLEMENTARY TEST X

	(a)	(b)	(c)	(d)	(e)
A.	39	19	37	39	37
B.	36	36	29	25	26
C.	18	28	34	37	15
D.	39	18	14	28	39
E.	17	12	19	39	16
F.	29	17	19	27	29
G.	27	35	26	29	29
H.	33	25	16	22	22
I.	30	20	30	25	32
J.	20	20	21	34	24
K.	21	33	30	36	31
L.	21	30	32	38	32
M.	31	21	27	33	35
N.	31	23	23	40	27
O.	24	24	26	31	34
P.	22	32	34	33	25
Q.	36	37	45	22	37
R.	20	33	35	32	21
S.	21	36	20	31	26
T.	25	28	22	23	23

SUPPLEMENTARY TEST Y

	(a)	(b)	(c)	(d)	(e)
1.	42	24	17	68	28
2.	83	18	39	66	46
3.	44	18	59	76	26
4.	87	79	57	15	38
5.	22	11	16	47	77
6.	27	86	66	17	39
7.	45	25	47	78	29
8.	69	29	19	33	35
9.	84	69	39	67	59
10.	89	27	38	37	75
11.	29	35	26	58	67
12.	88	56	19	48	74
13.	28	36	37	19	48
14.	45	22	65	49	23
15.	51	20	60	20	22
16.	54	70	50	40	31
17.	21	30	20	30	31
18.	60	22	50	30	54
19.	33	26	21	23	42
20.	41	24	34	25	51
21.	62	32	53	41	31
22.	80	52	53	55	50
23.	43	33	40	63	32
24.	71	71	51	52	35
25.	62	34	61	68	52
26.	53	44	55	70	61
27.	41	30	90	104	92
28.	42	93	102	91	105
29.	103	56	57	45	33
30.	32	85	75	108	114
31.	96	116	97	50	62
32.	115	117	104	86	81

SUPPLEMENTARY TEST Z

	(a)	(b)	(c)	(d)	(e)
A.	6 r.1	11 r.1	9 r.1	11 r.1	7 r.1
B.	9 r.1	6 r.1	11 r.1	9 r.1	11 r.1
C.	9 r.1	9 r.1	12 r.1	6 r.1	8 r.1
D.	2 r.2	2 r.4	4 r.4	8 r.2	6 r.2
E.	3 r.2	6 r.2	5 r.5	3 r.6	2 r.6
F.	7 r.2	6 r.4	6 r.6	8 r.6	3 r.3
G.	4 r.2	5 r.5	3 r.4	8 r.4	8 r.8
H.	4 r.7	6 r.7	4 r.6	8 r.6	2 r.8
I.	3 r.5	8 r.5	3 r.7	3 r.4	3 r.6
J.	2 r.5	4 r.6	4 r.4	6 r.7	2 r.3
K.	7 r.3	8 r.5	4 r.3	8 r.3	4 r.5
L.	6 r.3	5 r.6	7 r.8	2 r.7	2 r.6
M.	4 r.5	2 r.5	4 r.6	5 r.8	2 r.6
N.	7 r.7	6 r.4	7 r.4	9 r.7	3 r.7
O.	2 r.4	9 r.4	6 r.6	9 r.3	9 r.2
P.	7 r.5	8 r.3	3 r.8	7 r.5	7 r.4
Q.	6 r.6	7 r.2	3 r.5	6 r.5	4 r.8
R.	7 r.4	2 r.5	9 r.6	11 r.3	7 r.6
S.	7 r.5	6 r.5	2 r.7	10 r.3	8 r.4
T.	2 r.5	5 r.2	5 r.6	12 r.2	3 r.4
U.	3 r.3	8 r.4	5 r.7	6 r.8	8 r.7

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BY

FRED J. SCHONELL, Ph.D., D.Lit.

Professor of Education, University College of Swansea

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